Lecture 11

Learning to Walk

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Peking University

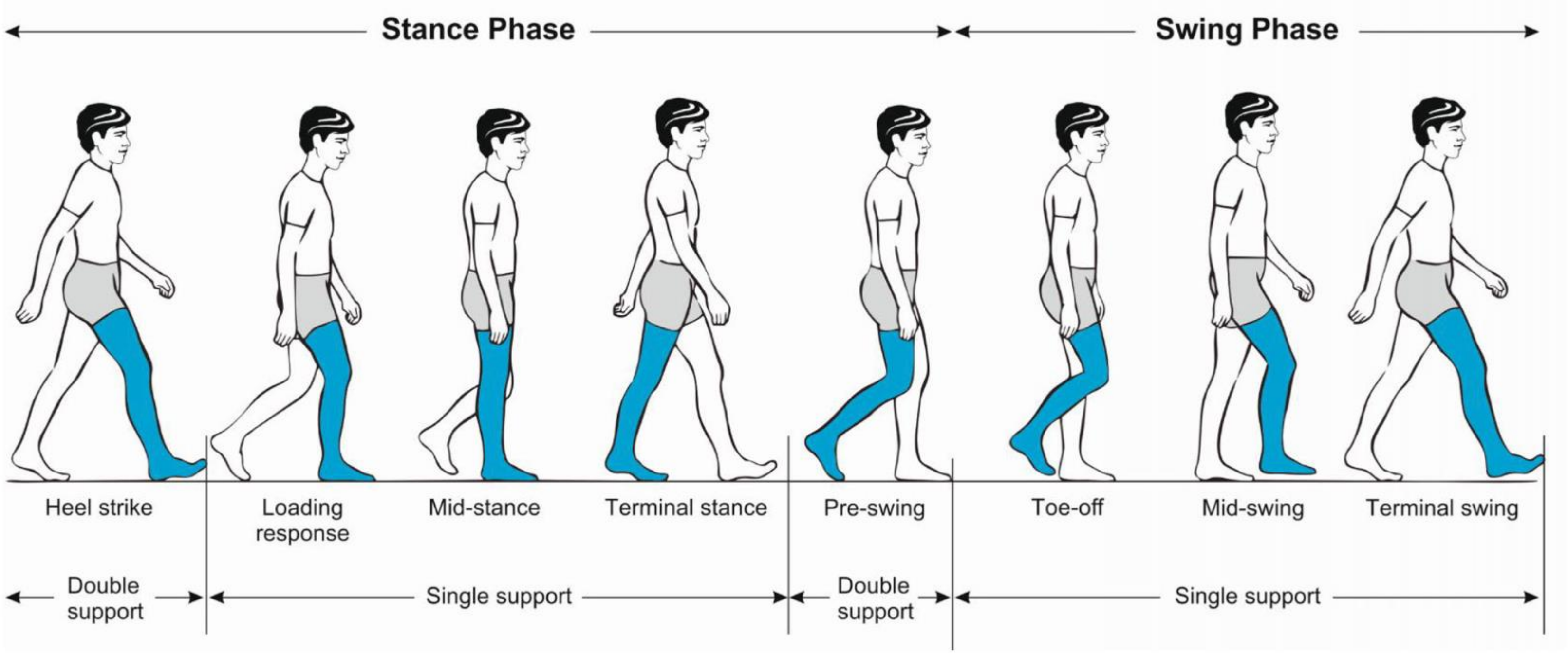
GAMES105 课程交流

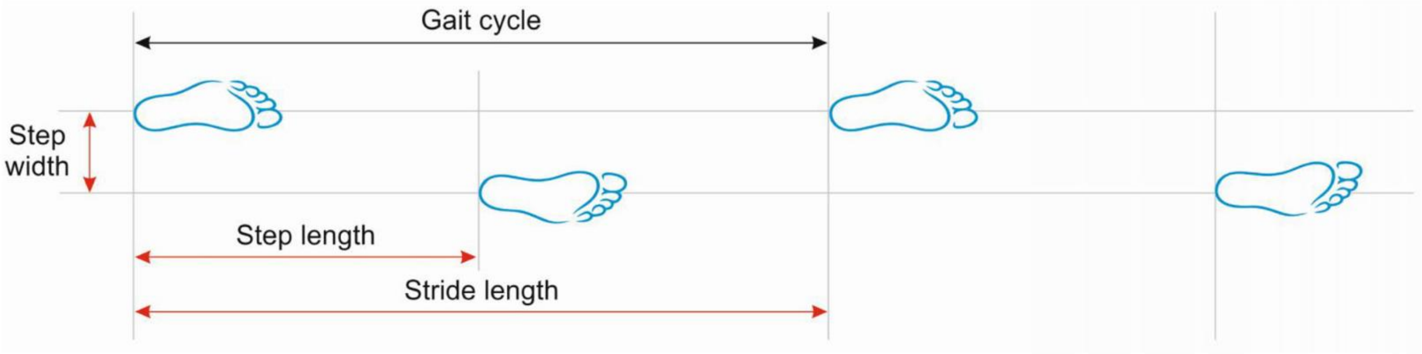
VCL @ PKU

Outline

- Walking and Dynamic Balance
- Simplified Models
  - ZMP (Zero-Moment Point)
  - Inverted Pendulum
  - SIMBICON

Walking

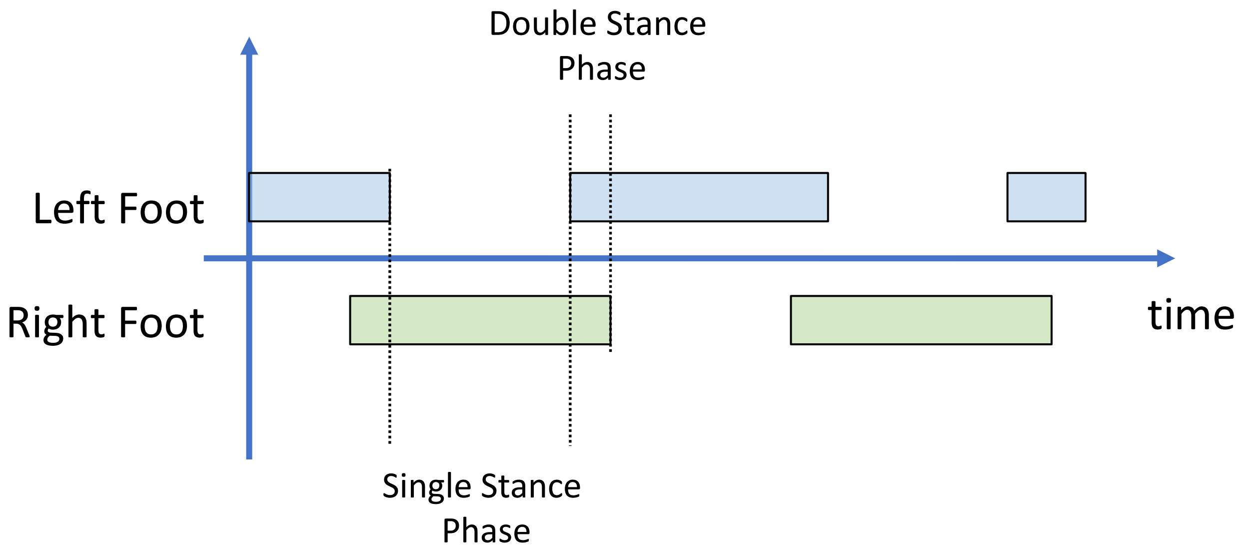




phases of a walking gait cycle

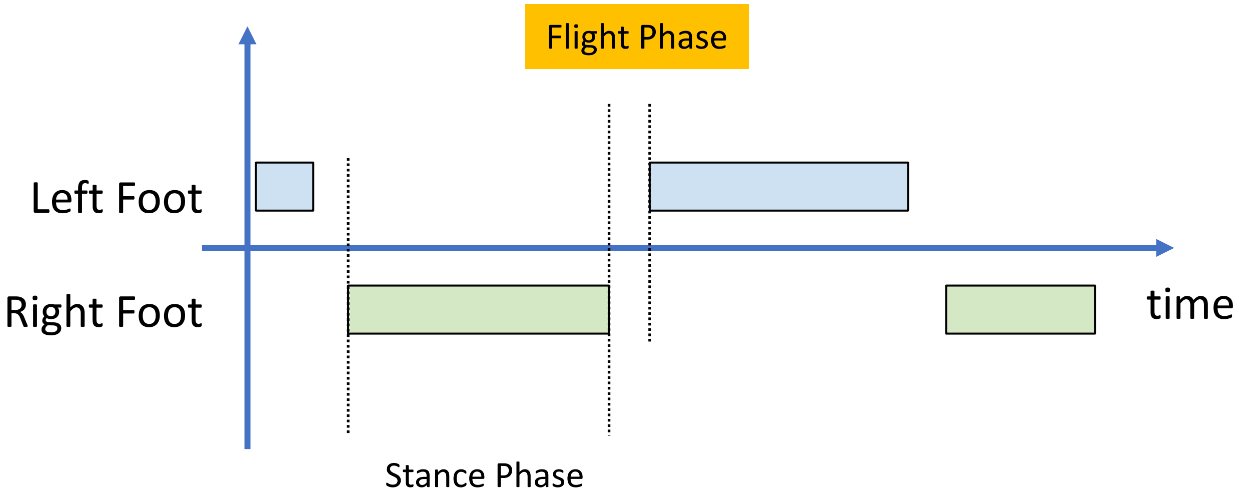
Pirker and Katzenschlager 2017.
Gait disorders in adults and the elderly.

Walking



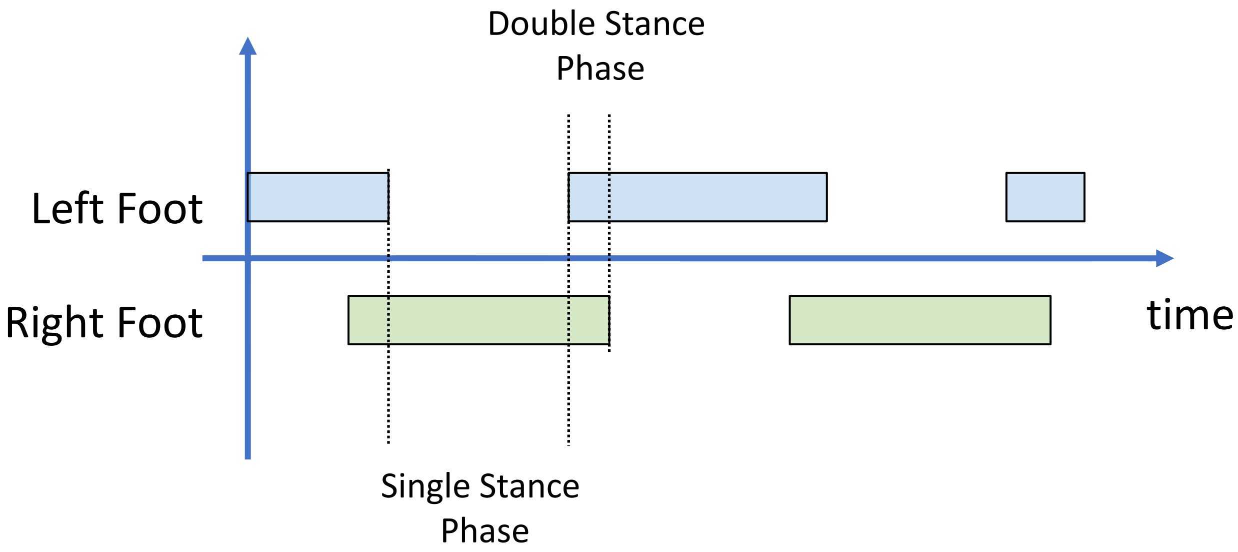
Walking: move without *loss of contact*, or flight phases

Walking

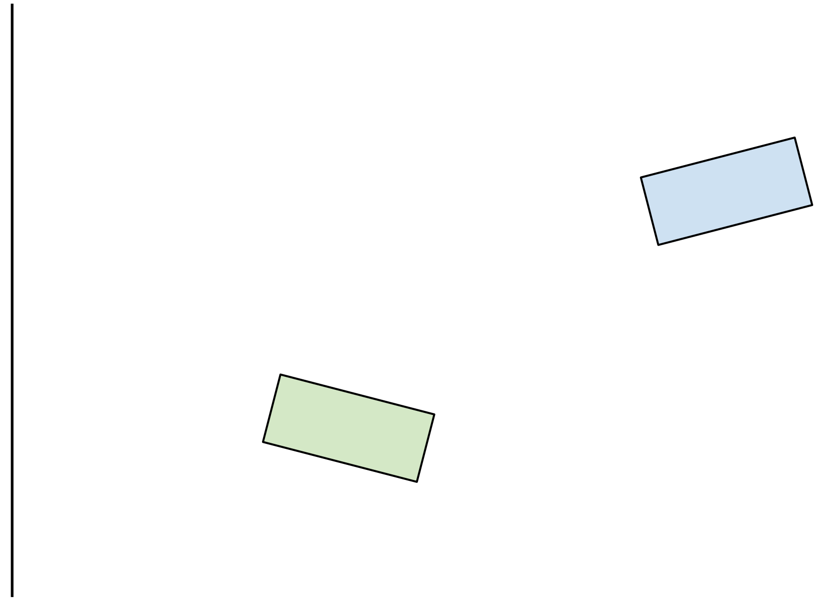


Running

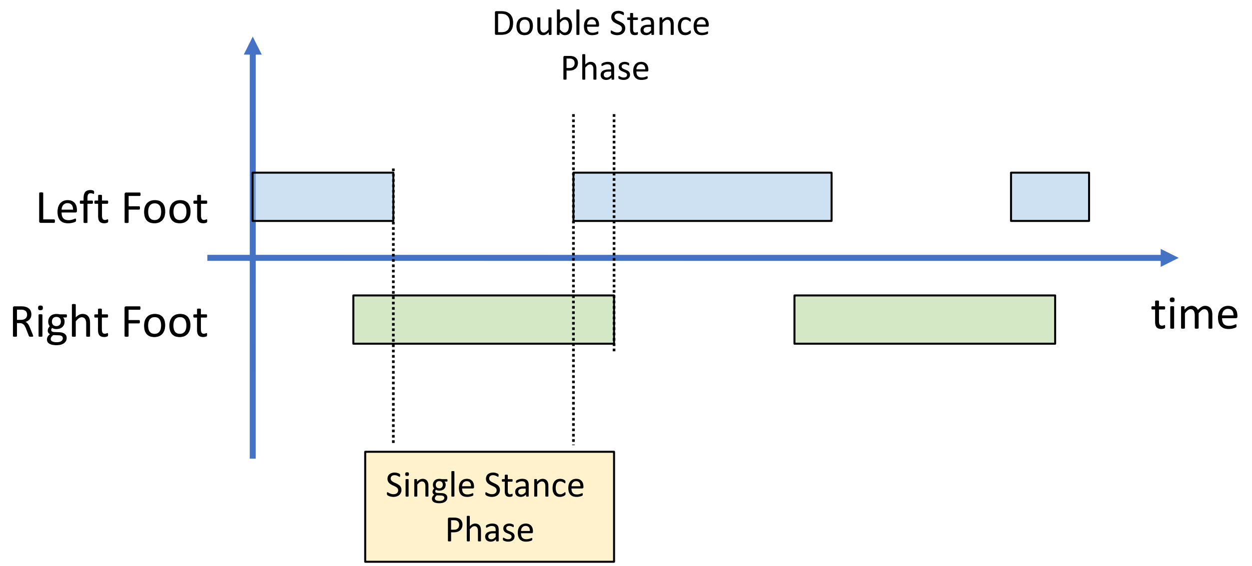
Walking with Static Balance



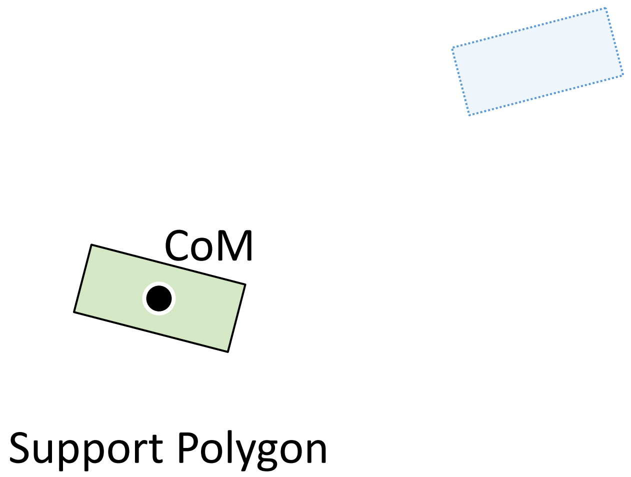
Walking: move without *loss of contact*, or flight phases



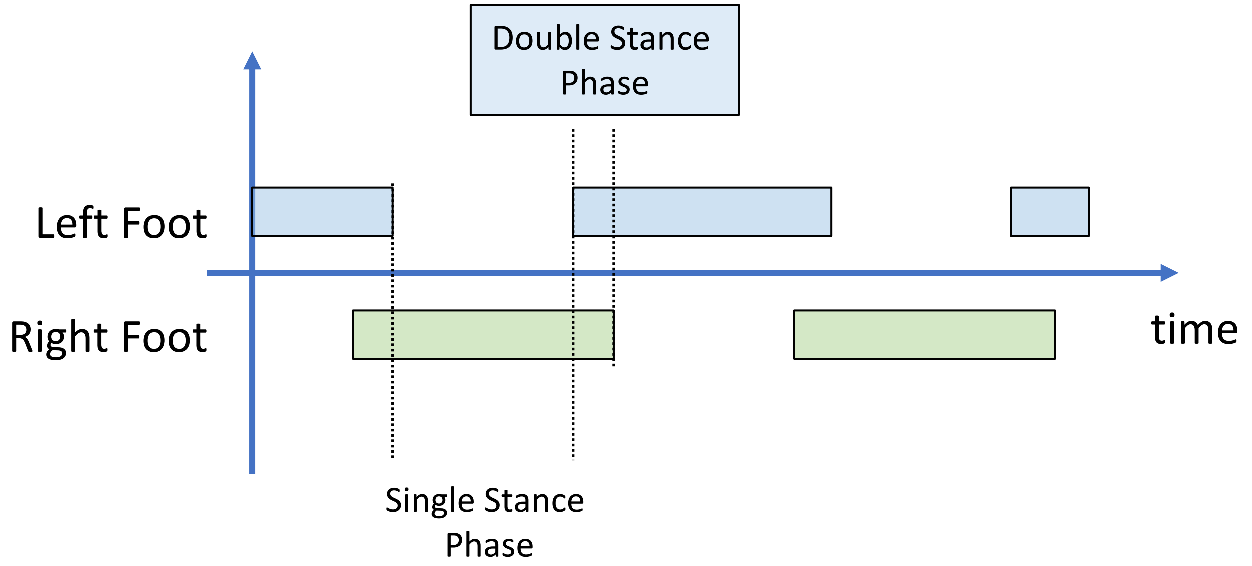
Walking with Static Balance



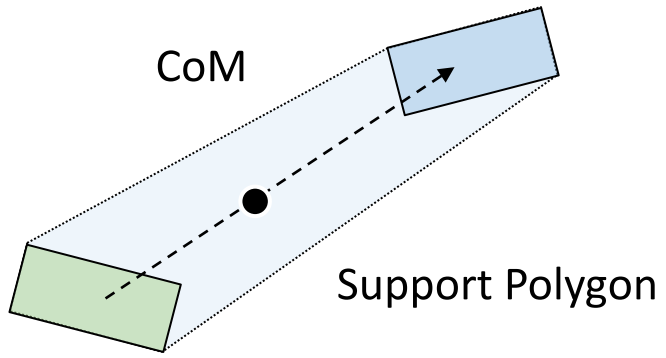
Walking: move without *loss of contact*, or flight phases



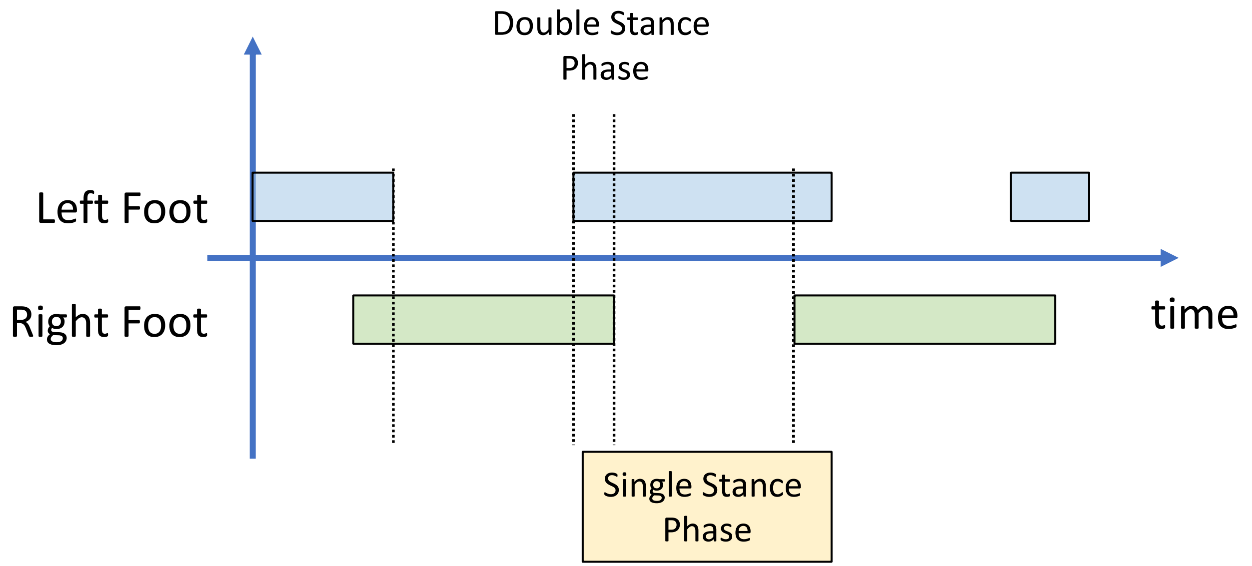
Walking with Static Balance

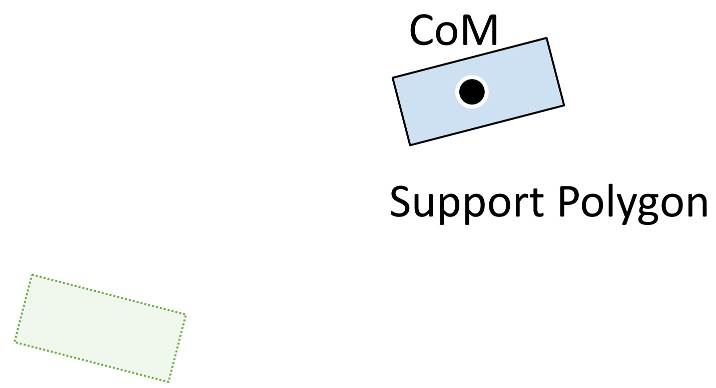


Walking: move without *loss of contact*, or flight phases



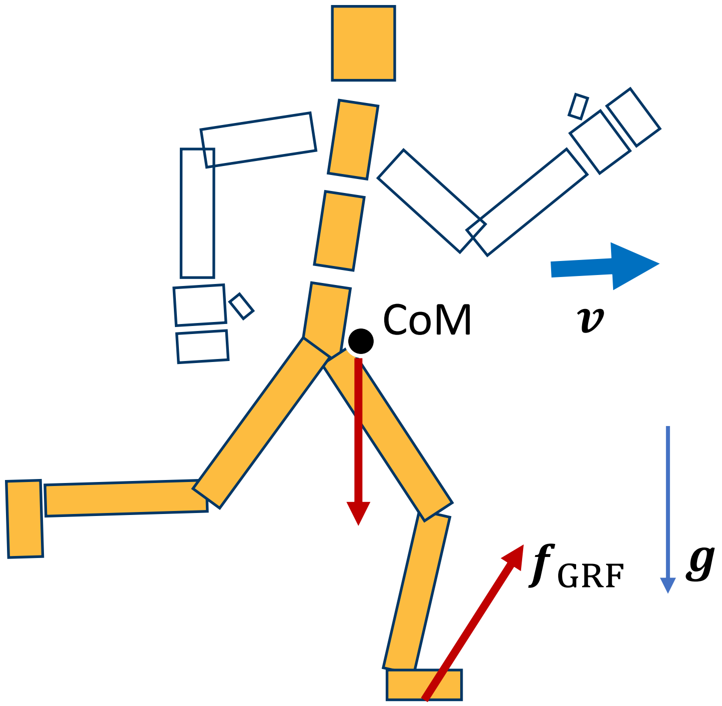
Walking with Static Balance





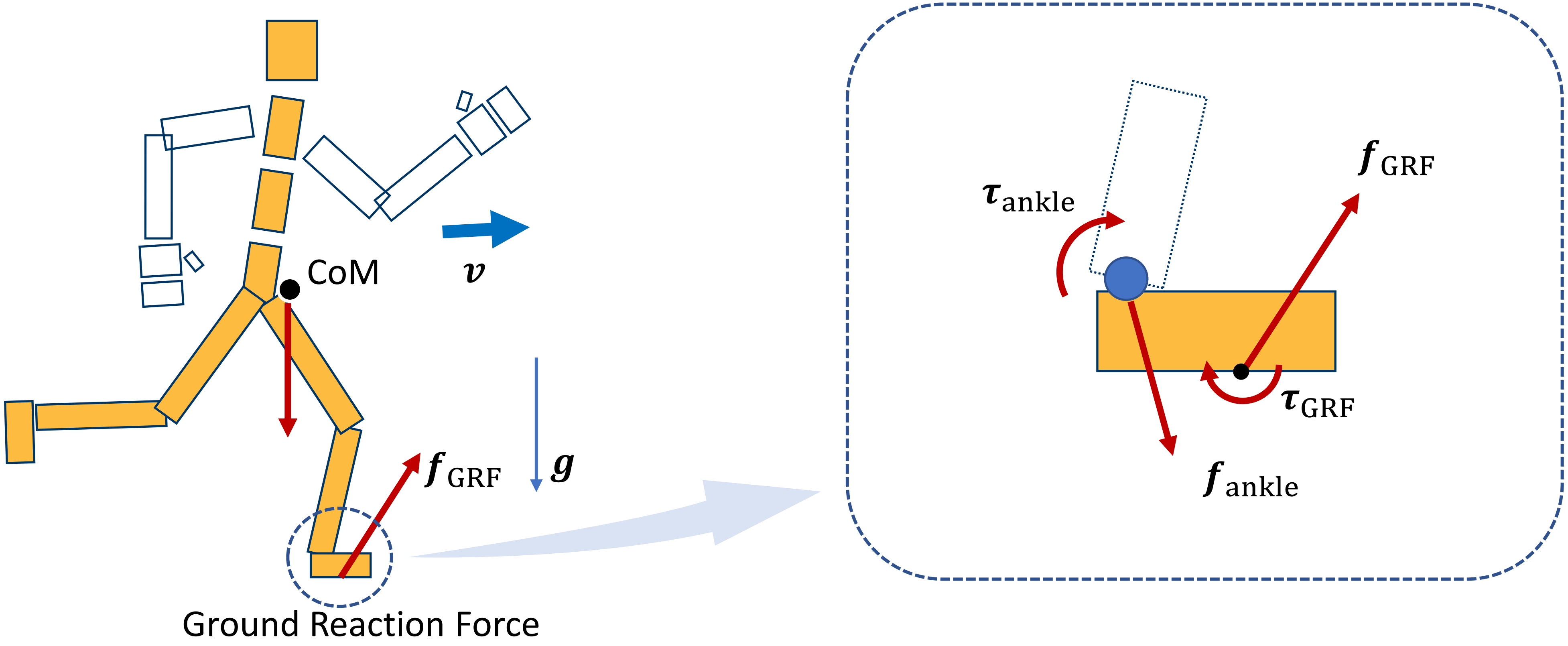
Walking: move without *loss of contact*, or flight phases

Zero-Moment Point (ZMP)

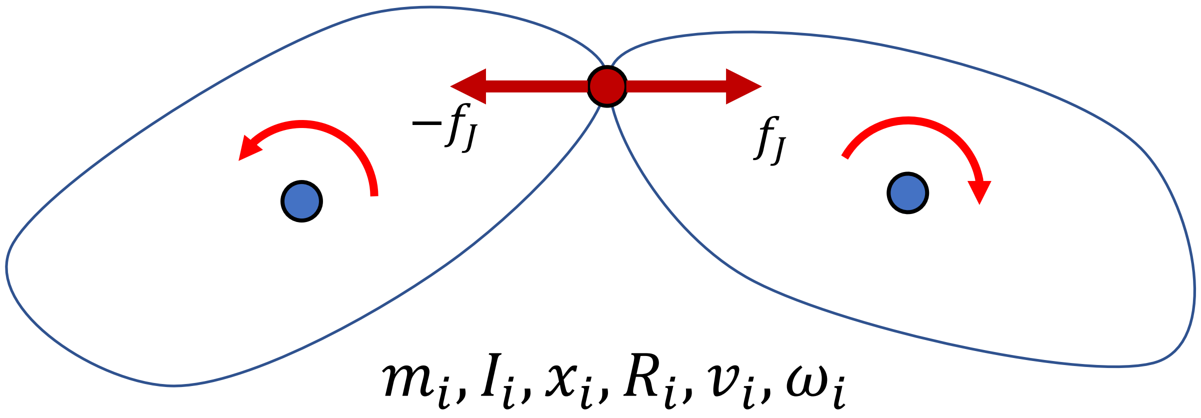


Ground Reaction Force

Zero-Moment Point (ZMP)

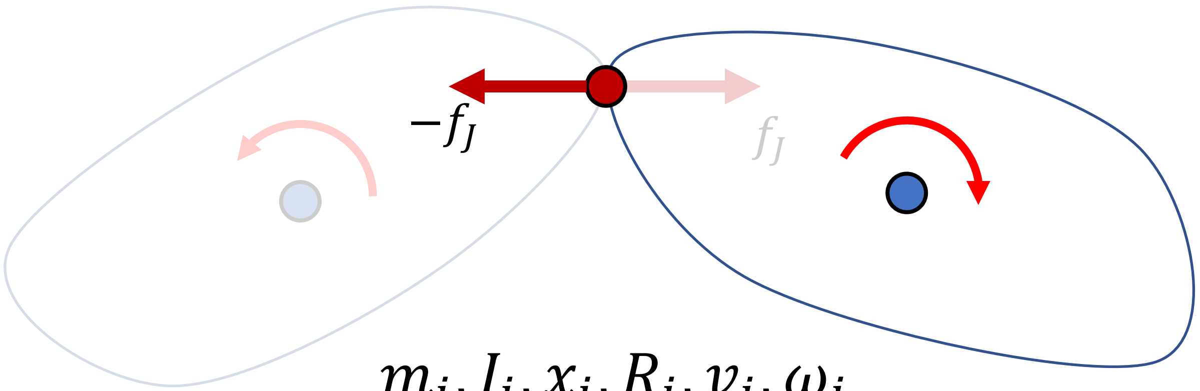


Recall: A System of Links and Joints



$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{f}_J$$

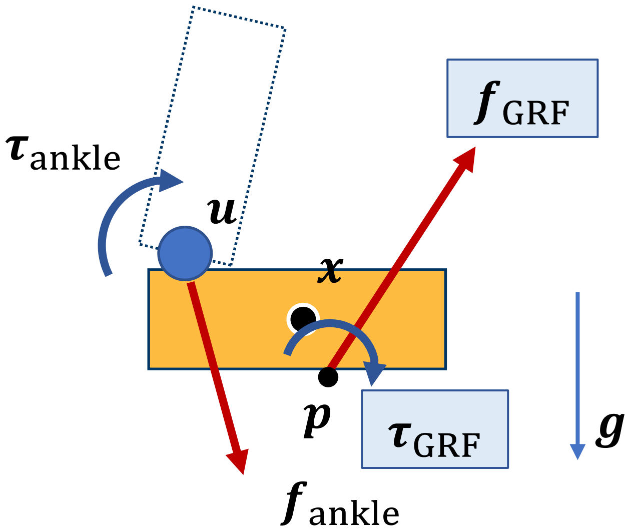
Recall: A System of Links and Joints



$m_i, I_i, x_i, R_i, v_i, \omega_i$

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{f}_J$$

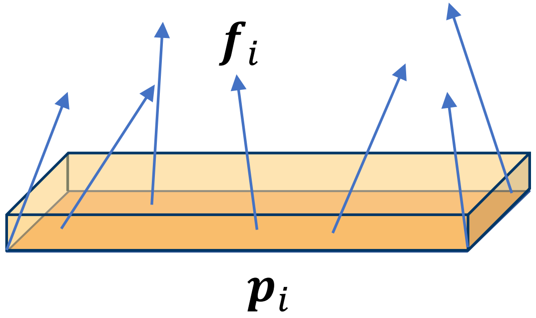
Zero-Moment Point (ZMP)



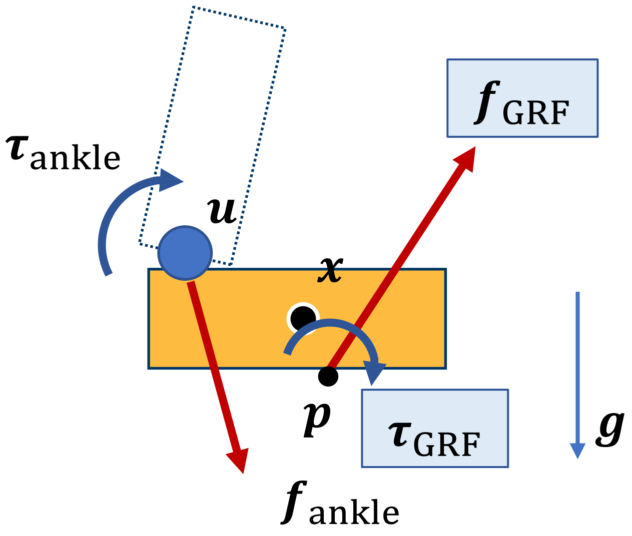
normal force

$$\boldsymbol{f}_i = \boldsymbol{f}_i^y + \boldsymbol{f}_i^{xz}$$

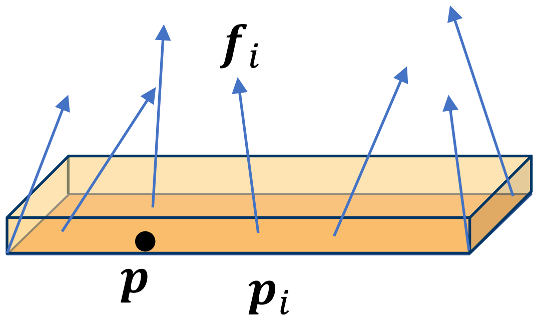
friction



Zero-Moment Point (ZMP)



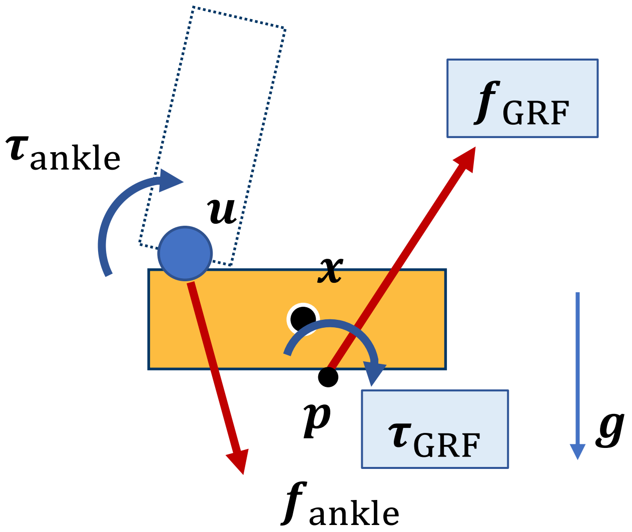
$$\boldsymbol{f}_i = \boldsymbol{f}_i^y + \boldsymbol{f}_i^{xz}$$



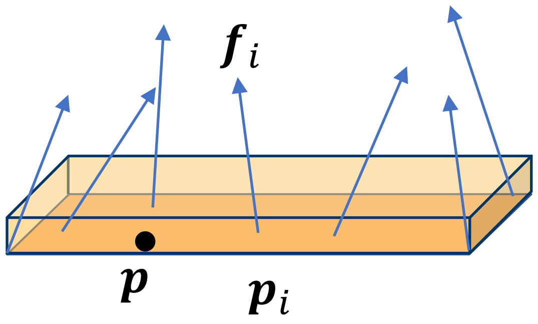
$$\boldsymbol{f}_{\text{GRF}} = \sum_i \boldsymbol{f}_i$$

$$\boldsymbol{\tau}_{\text{GRF}} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i$$

Zero-Moment Point (ZMP)



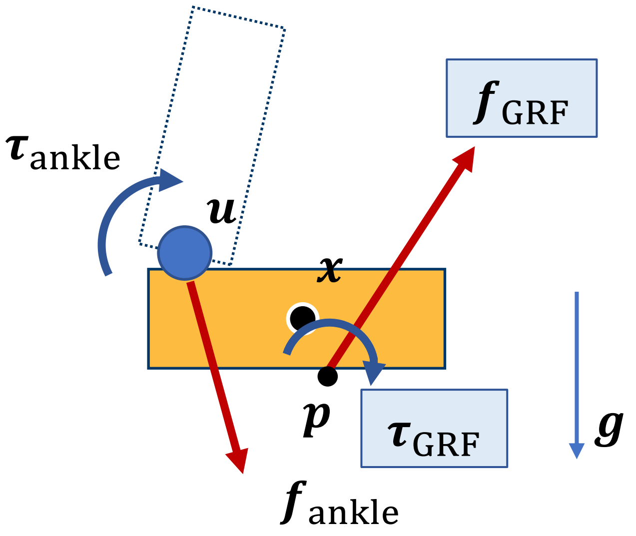
$$f_i = f_i^y + f_i^{xz}$$



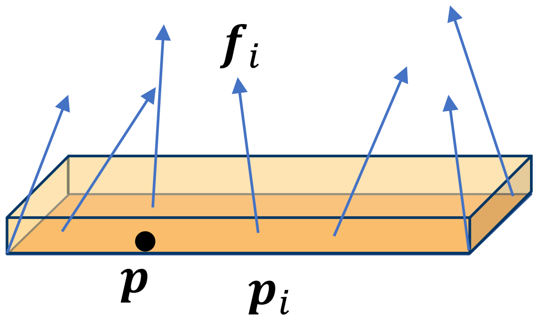
$$\tau_{\text{GRF}} = \sum_i (p_i - p) \times f_i$$

Assuming the ground is flat and level
so $p_i - p$ is always in the horizontal plane

Zero-Moment Point (ZMP)



$$\boldsymbol{f}_i = \boldsymbol{f}_i^y + \boldsymbol{f}_i^{xz}$$



$$\boldsymbol{\tau}_{\text{GRF}} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i$$

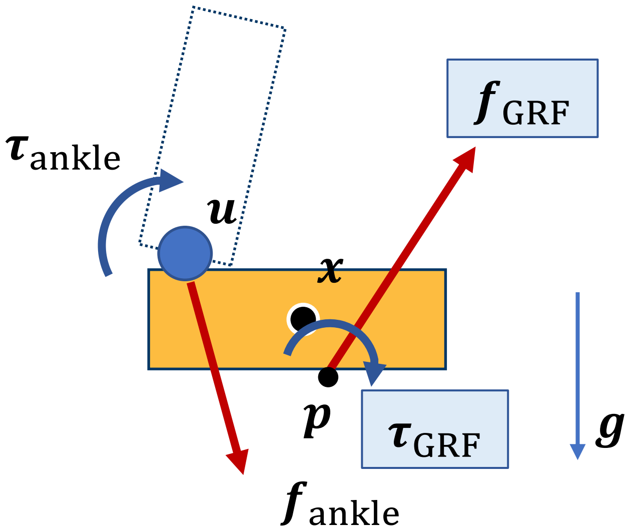
Assuming the ground is flat and level so $\boldsymbol{p}_i - \boldsymbol{p}$ is always in the horizontal plane

$$\boldsymbol{\tau}_{\text{GRF}} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \left(f_i^y + f_i^{xz}\right)$$

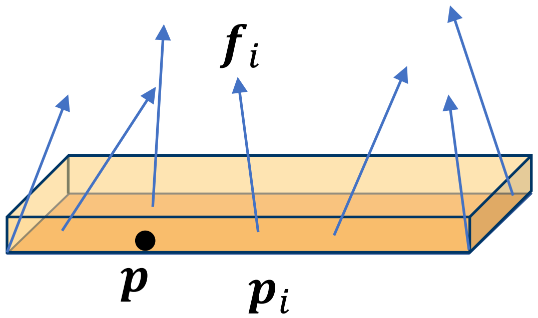
horizontal　　　　vertical

$$= \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times f_i^y + \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times f_i^{xz}$$

Zero-Moment Point (ZMP)



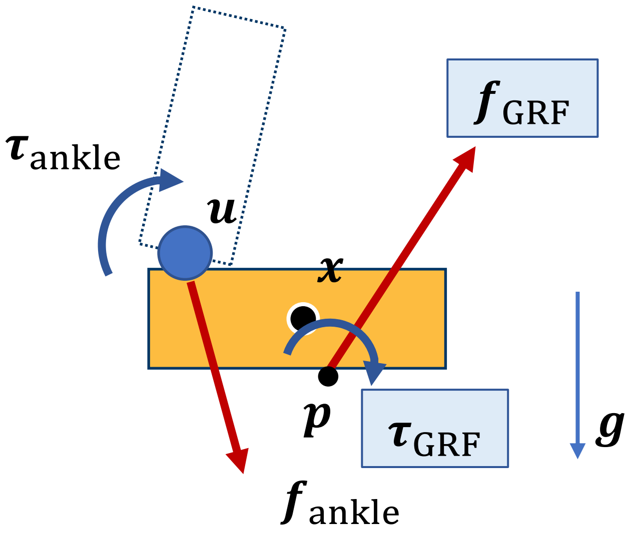
$$\boldsymbol{f}_i = \boldsymbol{f}_i^y + \boldsymbol{f}_i^{xz}$$



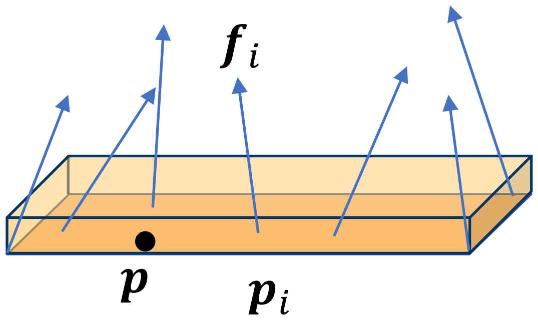
$$\boldsymbol{\tau}_{\text{GRF}}^y = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^{xz}$$

$$\boldsymbol{\tau}_{\text{GRF}}^{xz} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^y$$

Zero-Moment Point (ZMP)



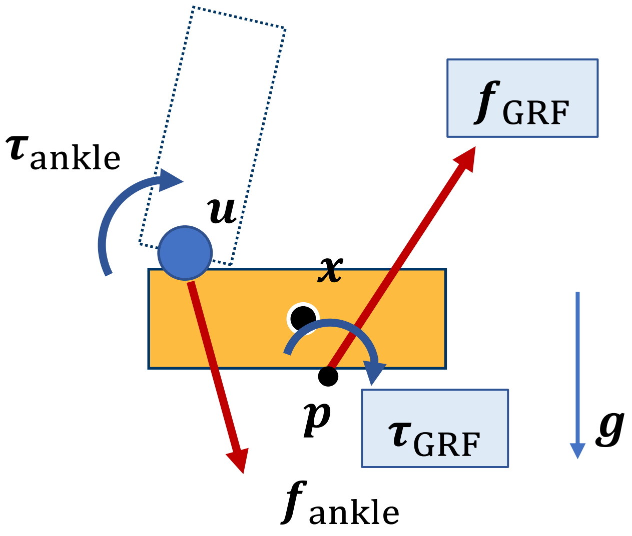
$$\boldsymbol{f}_i = \boldsymbol{f}_i^y + \boldsymbol{f}_i^{xz}$$



$$\boldsymbol{\tau}_{\text{GRF}}^{xz} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^y$$

$$= \sum_i \boldsymbol{p}_i \times \boldsymbol{f}_i^y - \boldsymbol{p} \times \left(\sum_i f_i^y \right) \boldsymbol{y}$$

Zero-Moment Point (ZMP)



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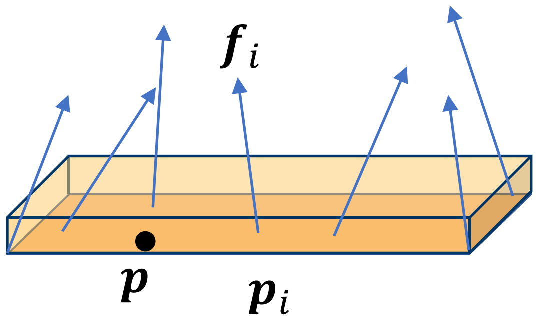
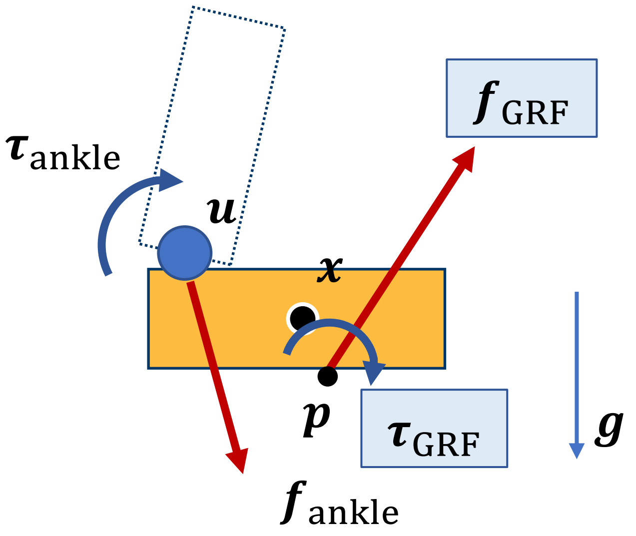


$$\boldsymbol{\tau}_{\text{GRF}}^{xz} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^y$$

$$= \sum_i \boldsymbol{p}_i \times \boldsymbol{f}_i^y - \boldsymbol{p} \times \left(\sum_i f_i^y \right) \boldsymbol{y}$$

Can we find $\boldsymbol{p}$ such that $\boldsymbol{\tau}_{\text{GRF}}^{xz} = 0$?

Zero-Moment Point (ZMP)



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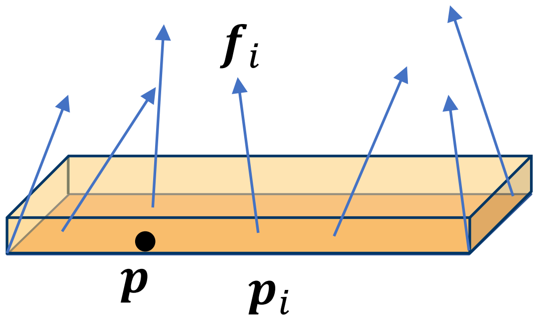
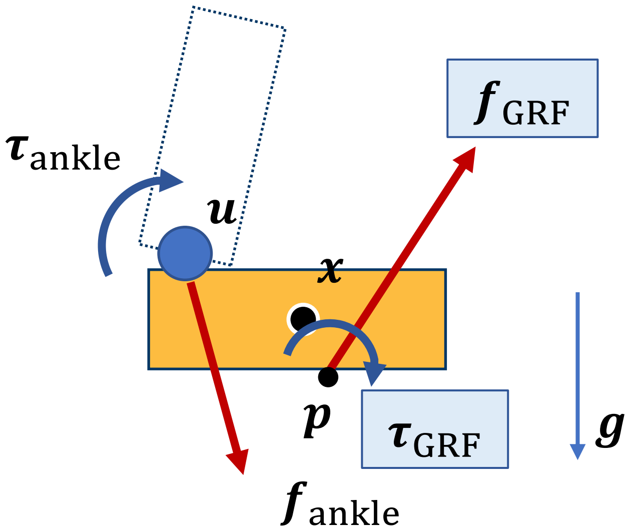


$$\boldsymbol{\tau}_{\text{GRF}}^{xz} = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^y$$

$$= \sum_i \boldsymbol{p}_i \times \boldsymbol{f}_i^y - \boldsymbol{p} \times \left(\sum_i f_i^y \right) \boldsymbol{y}$$

Center of Pressure $\boldsymbol{p} = \frac{\sum_i \boldsymbol{p}_i f_i^y}{\sum_i f_i^y}$ ⇨ $\boldsymbol{\tau}_{\text{GRF}}^{xz} = 0$

Zero-Moment Point (ZMP)



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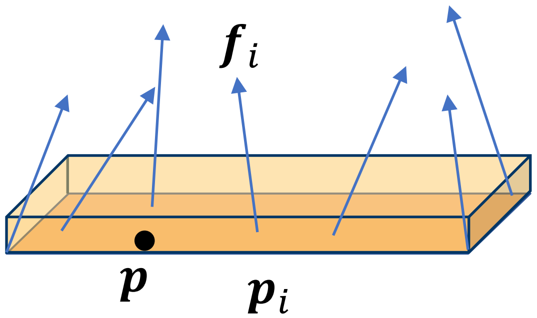


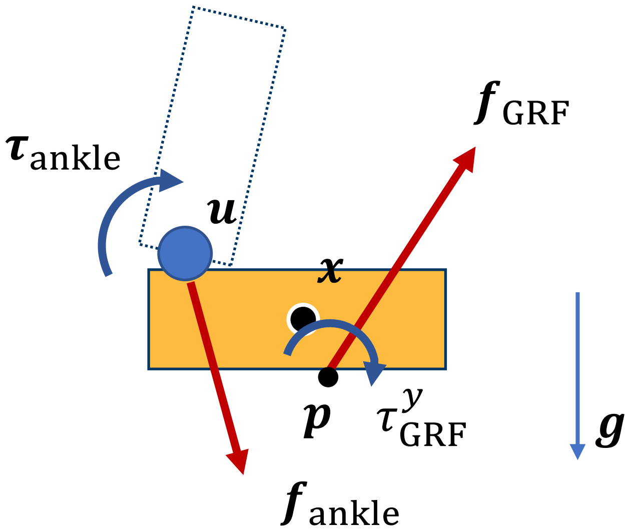
Center of Pressure

$$\boldsymbol{p} = \frac{\sum_i \boldsymbol{p}_i f_i^y}{\sum_i f_i^y}$$

$$\boldsymbol{f}_{\text{GRF}} = \sum_i \boldsymbol{f}_i$$

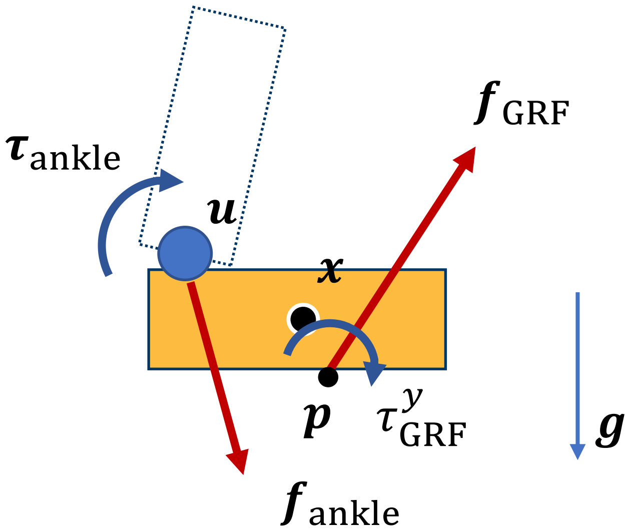
$$\boldsymbol{\tau}_{\text{GRF}} = \tau_{\text{GRF}}^y = \sum_i (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i^{xz}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

Zero-Moment Point (ZMP)



The position of $\boldsymbol{p}$ is not known, but we assume

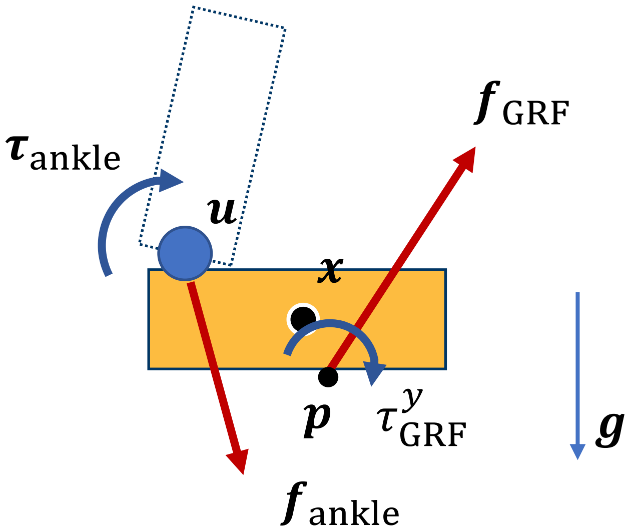
$$\tau_{\text{GRF}}^{xz} = \mathbf{0}$$

So

$$\boldsymbol{\tau}_{\text{GRF}} = \tau_{\text{GRF}}^{y}$$

The foot should not move in a **stance phase**

Zero-Moment Point (ZMP)

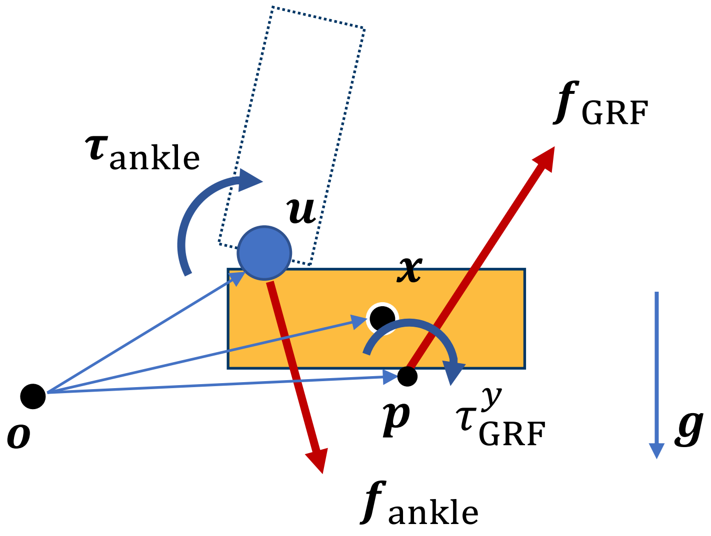


The foot should not move in a **stance phase**

Static Equilibrium:

$$\boldsymbol{f}_{\text{ankle}} + \boldsymbol{f}_{\text{GRF}} + m\boldsymbol{g} = \boldsymbol{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

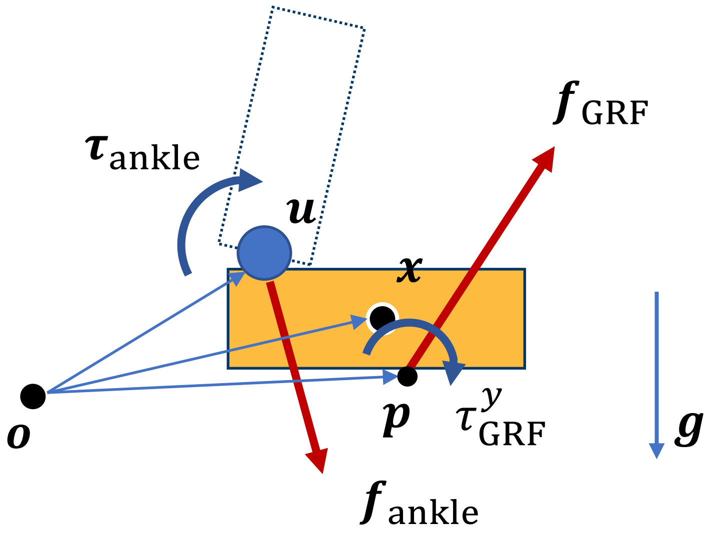
Static Equilibrium:

$$\boldsymbol{f}_{\text{ankle}} + \boldsymbol{f}_{\text{GRF}} + m\boldsymbol{g} = \mathbf{0}$$

The moment around a reference point $\boldsymbol{o}$:

$$(\boldsymbol{u} - \boldsymbol{o}) \times \boldsymbol{f}_{\text{ankle}} + (\boldsymbol{p} - \boldsymbol{o}) \times \boldsymbol{f}_{\text{GRF}} + (\boldsymbol{x} - \boldsymbol{o}) \times m\boldsymbol{g}$$
$$+\tau^{y}_{\text{GRF}} + \boldsymbol{\tau}_{\text{ankle}} = \mathbf{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

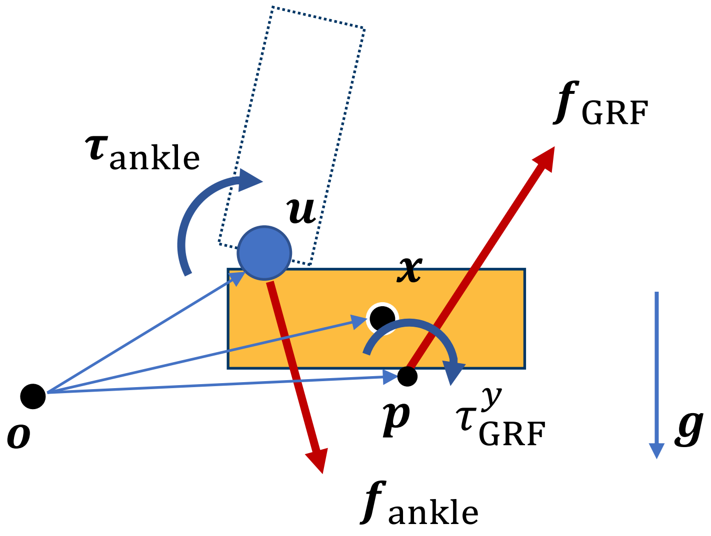
The moment around a reference point $\boldsymbol{o}$:

$$(\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}+(\boldsymbol{p}-\boldsymbol{o})\times\boldsymbol{f}_{\text{GRF}}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}$$
$$+\tau_{\text{GRF}}^{y}+\boldsymbol{\tau}_{\text{ankle}}=\mathbf{0}$$

Horizontal components (moment projected onto xz plane):

$$\left((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\right)^{xz}+\left((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\right)^{xz}$$
$$+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}_{\text{ankle}}^{xz}=\mathbf{0}$$

Zero-Moment Point (ZMP)



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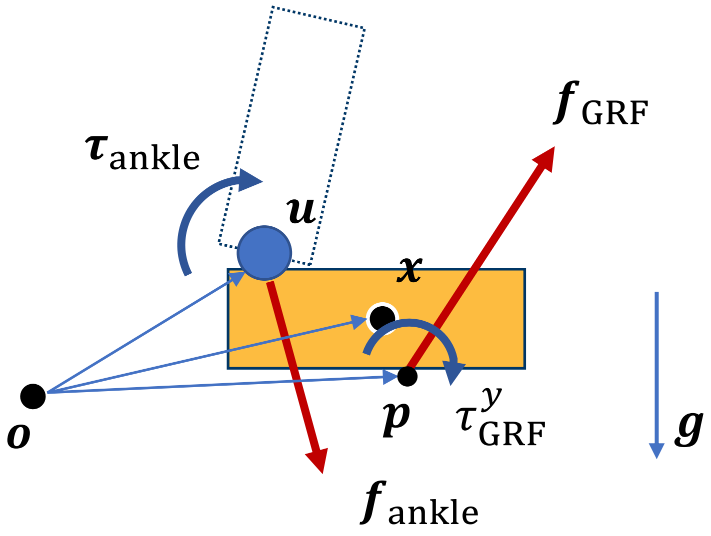
$$(\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}+(\boldsymbol{p}-\boldsymbol{o})\times\boldsymbol{f}_{\text{GRF}}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}$$
$$+\tau^{y}_{\text{GRF}}+\boldsymbol{\tau}_{\text{ankle}}=\mathbf{0}$$

Horizontal components (moment projected onto xz plane):

$$\big((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\big)^{xz}+\big((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\big)^{xz}$$
$$+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}^{xz}_{\text{ankle}}=\mathbf{0}$$

We can solve this equation to find $\boldsymbol{p}$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

$\boldsymbol{p}$ is called Zero-Moment Point (ZMP) because it makes

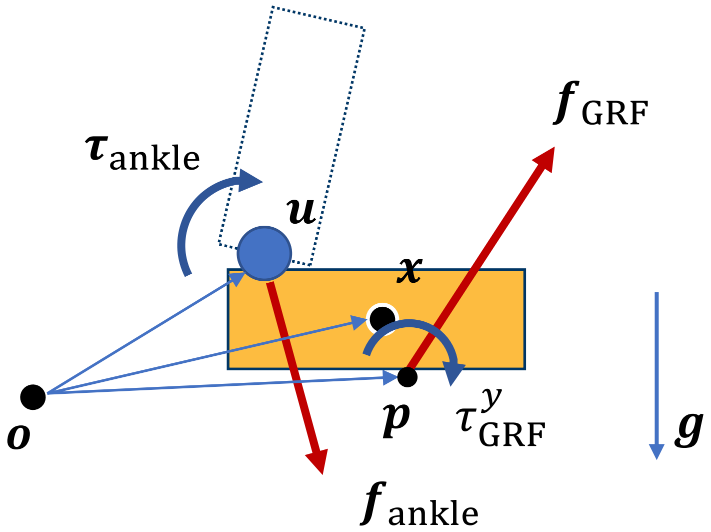
$$\tau_{\text{GRF}}^{xz} = \mathbf{0}$$

and the horizontal moment

$$\big((\boldsymbol{u}-\boldsymbol{o}) \times \boldsymbol{f}_{\text{ankle}}\big)^{xz} + \big((\boldsymbol{p}-\mathbf{o}) \times \boldsymbol{f}_{\text{GRF}}\big)^{xz}$$

$$+ (\boldsymbol{x}-\boldsymbol{o}) \times m\boldsymbol{g} + \boldsymbol{\tau}_{\text{ankle}}^{xz} = \mathbf{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

$\boldsymbol{p}$ is called Zero-Moment Point (ZMP) because it makes

$$\tau^{xz}_{\text{GRF}} = \mathbf{0}$$

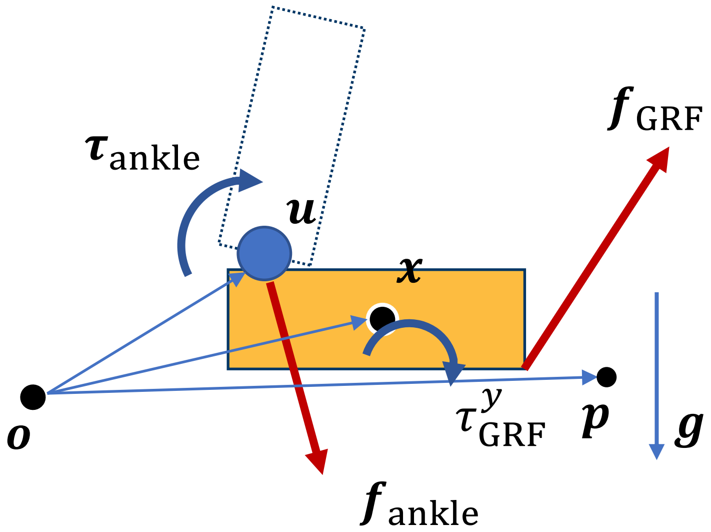
and the horizontal moment

$$\left((\boldsymbol{u}-\boldsymbol{o}) \times \boldsymbol{f}_{\text{ankle}}\right)^{xz} + \left((\boldsymbol{p}-\mathbf{o}) \times \boldsymbol{f}_{\text{GRF}}\right)^{xz}$$

$$+ (\boldsymbol{x}-\boldsymbol{o}) \times m\boldsymbol{g} + \boldsymbol{\tau}^{xz}_{\text{ankle}} = \mathbf{0}$$

Only when p is within the support polygon!

Zero-Moment Point (ZMP)



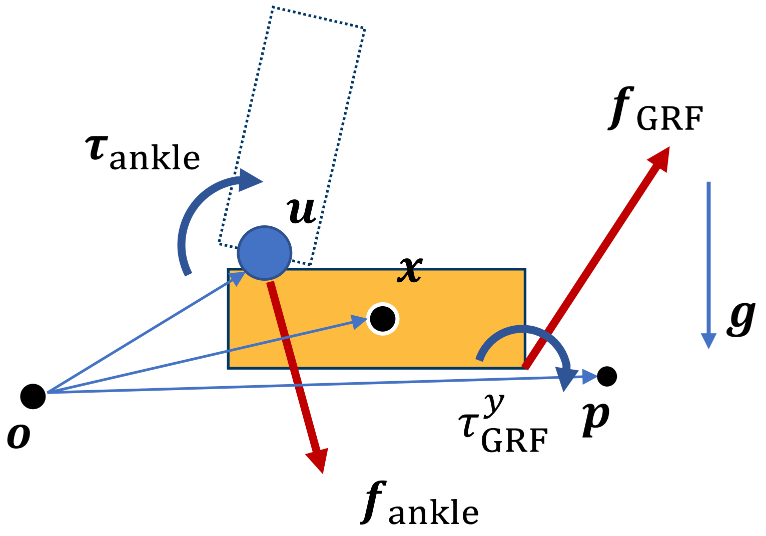
The foot should not move in a **stance phase**

If the solution of

$$\left((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\right)^{xz}+\left((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\right)^{xz}$$
$$+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}^{xz}_{\text{ankle}}=\mathbf{0}$$

$\boldsymbol{p}$ is outside the support polygon

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

If the solution of

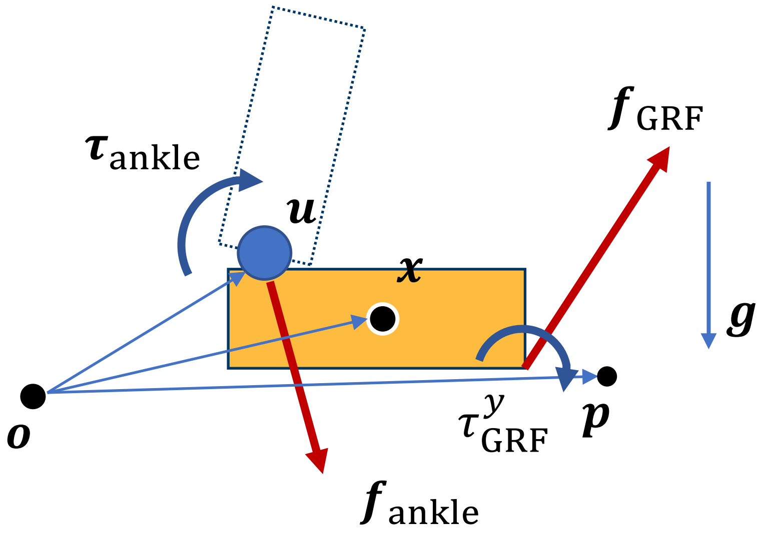
$$\left((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\right)^{xz}+\left((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\right)^{xz}$$
$$+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}_{\text{ankle}}^{xz}=\mathbf{0}$$

$\boldsymbol{p}$ is outside the support polygon

$\boldsymbol{p}$ could NOT be the center of pressure, because all the GRFs are applied within the polygon, so that

$$\tau_{\text{GRF}}^{xz}\neq\mathbf{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

If the solution of

$$\left((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\right)^{xz}+\left((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\right)^{xz}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}_{\text{ankle}}^{xz}=\mathbf{0}$$

$\boldsymbol{p}$ is outside the support polygon

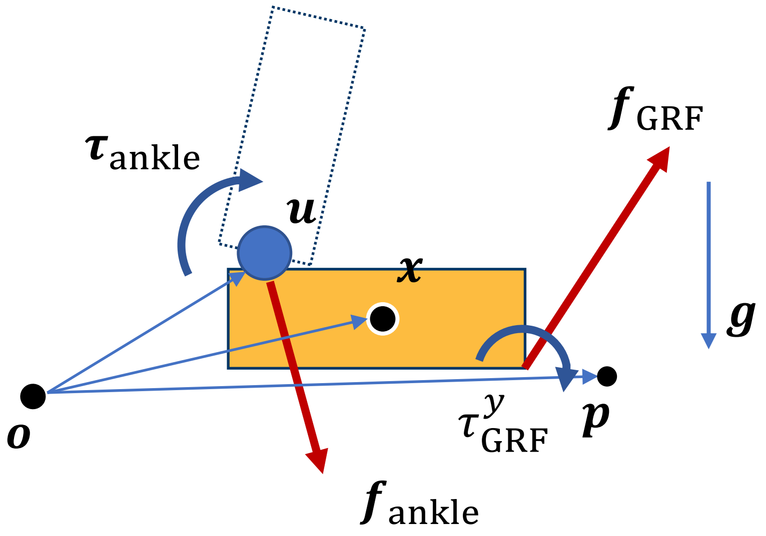
$\boldsymbol{p}$ could NOT be the center of pressure, because all the GRFs are applied within the polygon, so that

$$\tau_{\text{GRF}}^{xz}\neq\mathbf{0}$$

Or, if $\boldsymbol{p}'$ is the real center of pressure, we have

$$\left((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\right)^{xz}+\left((\boldsymbol{p}'-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\right)^{xz}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}_{\text{ankle}}^{xz}\neq\mathbf{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

If the solution of

$$\big((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\big)^{xz}+\big((\boldsymbol{p}-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\big)^{xz}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}^{xz}_{\text{ankle}}=\mathbf{0}$$

$\boldsymbol{p}$ is outside the support polygon

$\boldsymbol{p}$ could NOT be the center of pressure, because all the GRFs are applied within the polygon, so that

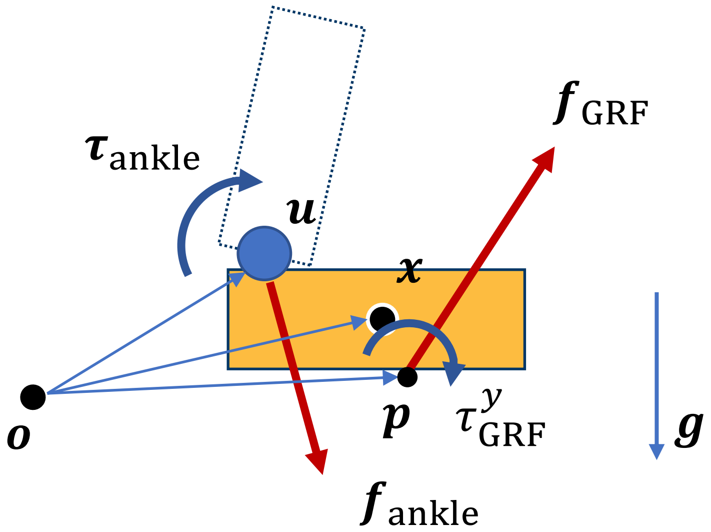
$$\tau^{xz}_{\text{GRF}}\neq\mathbf{0}$$

Or, if $\boldsymbol{p}'$ is the real center of pressure, we have

the foot will rotate…

$$\big((\boldsymbol{u}-\boldsymbol{o})\times\boldsymbol{f}_{\text{ankle}}\big)^{xz}+\big((\boldsymbol{p}'-\mathbf{o})\times\boldsymbol{f}_{\text{GRF}}\big)^{xz}+(\boldsymbol{x}-\boldsymbol{o})\times m\boldsymbol{g}+\boldsymbol{\tau}^{xz}_{\text{ankle}}\neq\mathbf{0}$$

Zero-Moment Point (ZMP)



The foot should not move in a **stance phase**

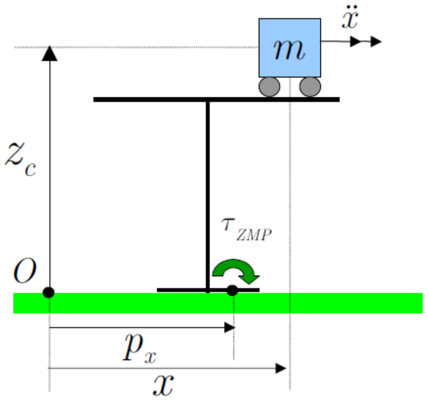
The existence of ZMP is an indication of dynamic balance

We can achieve balanced walking by controlling ZMP

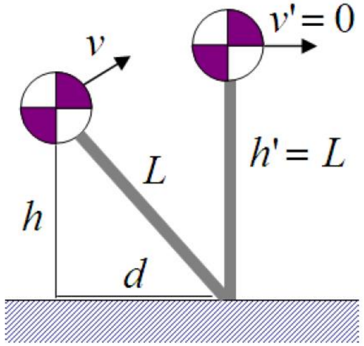
But how?

Simplified Models

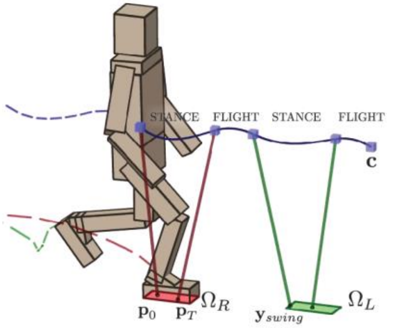
- Simplify humanoid / biped robot into an abstract model
  - Often consists of a CoM and a massless mechanism
  - Need to map the state of the robot to the abstract model
- Plan the control and movement of the model
  - Optimization
  - Dynamic programming
  - Optimal control
  - MPC
- Track the planned motion of the abstract model
  - Inverse Kinematics
  - Inverse Dynamics



[Kajita et al. 2003]

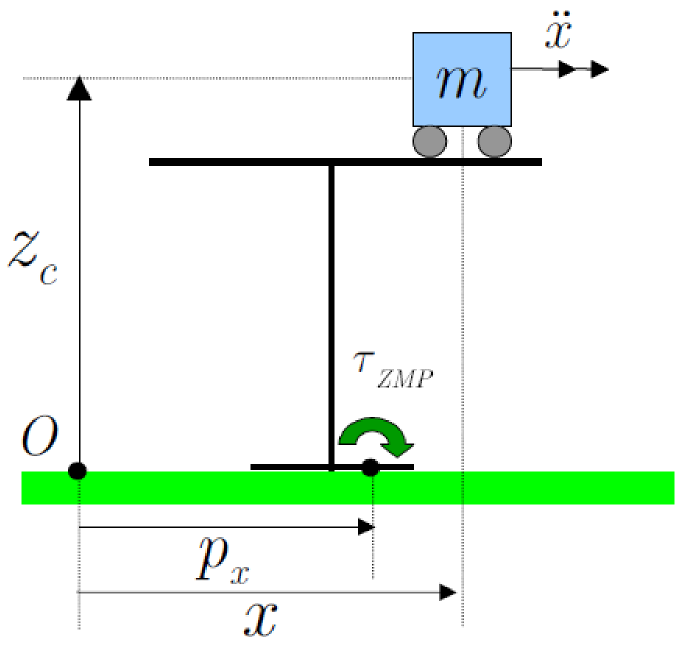


[Coros et al. 2010]



[Mordatch et al. 2010]

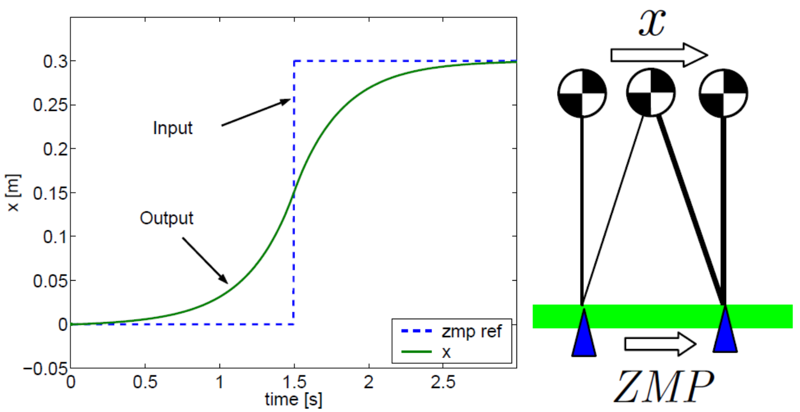
Example: ZMP-Guided Control



[Kajita et al. 2003]

Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point

Shuuji KAJITA, Fumio KANEHIRO, Kenji KANEKO, Kiyoshi FUJIWARA, Kensuke HARADA, Kazuhito YOKOI and Hirohisa HIRUKAWA



Example: ZMP-Guided Control

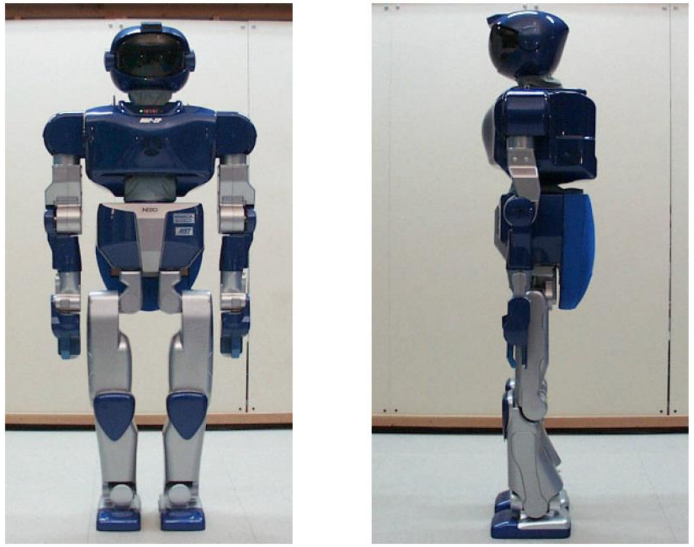
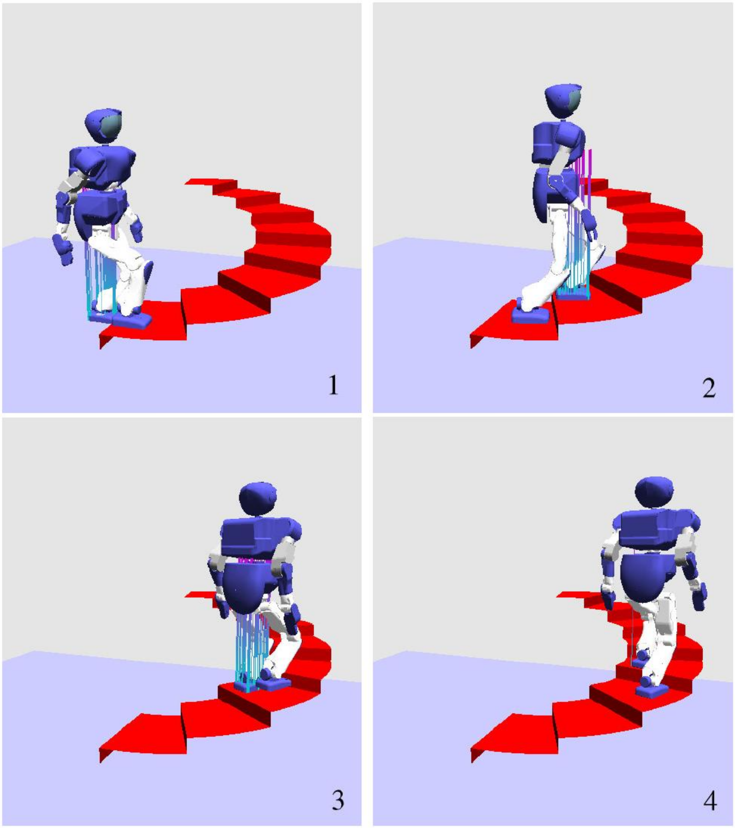
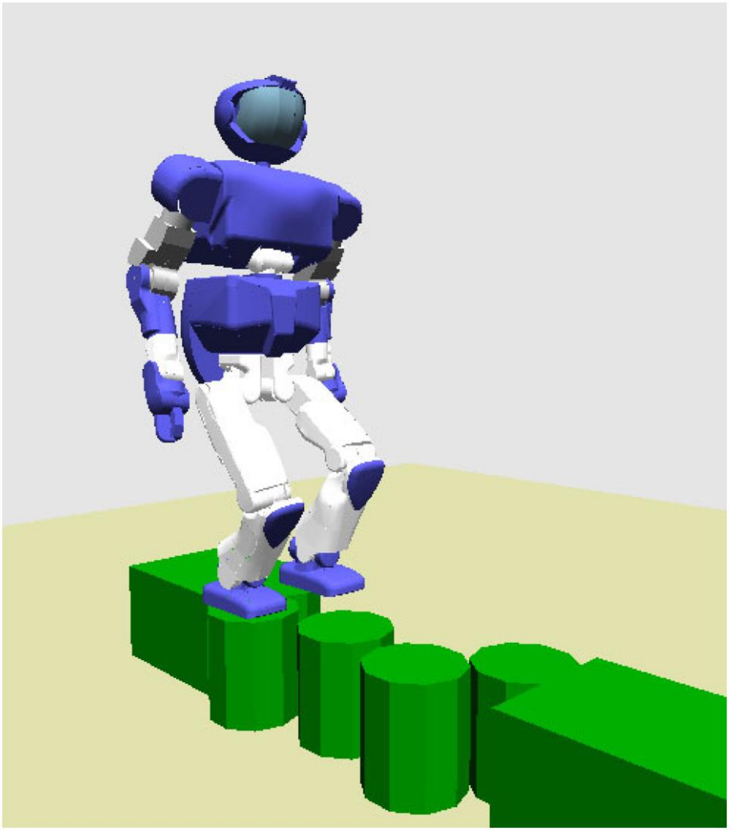


Figure 9: HRP-2 Prototype (HRP-2P)[22]





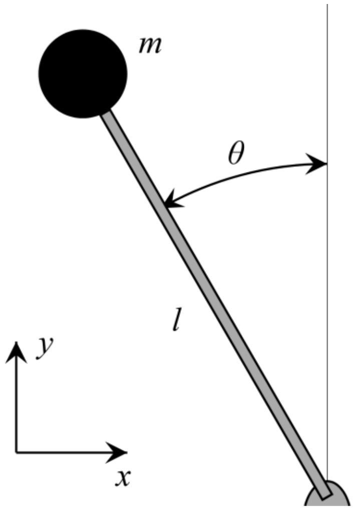
[Kajita et al. 2003]

ASIMO

Walking == Falling + Step Planning

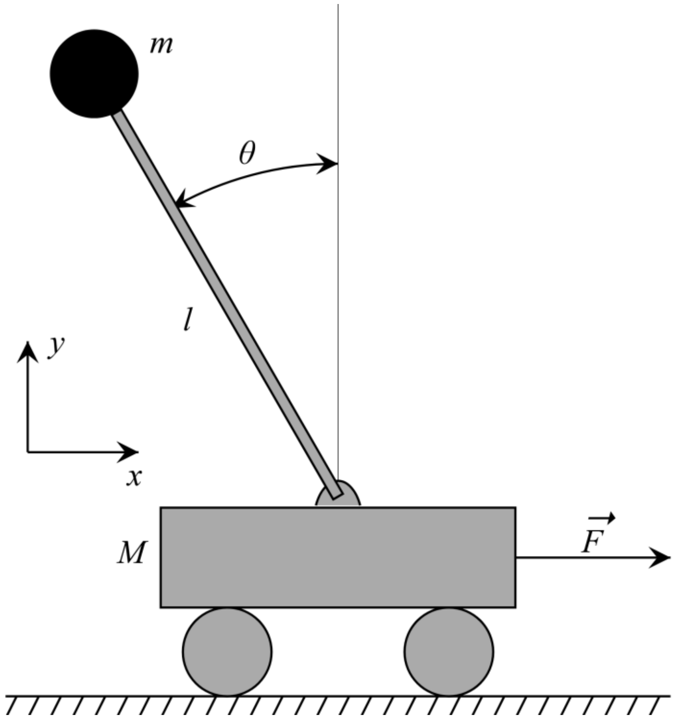
Passive Dynamic Walker

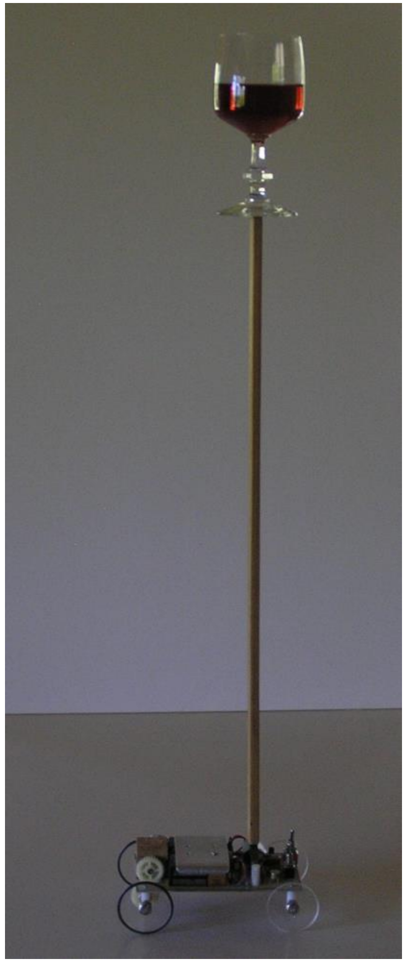
Inverted Pendulum Model (IPM)



$$\ddot{\theta} = \frac{g}{\ell} \sin \theta$$

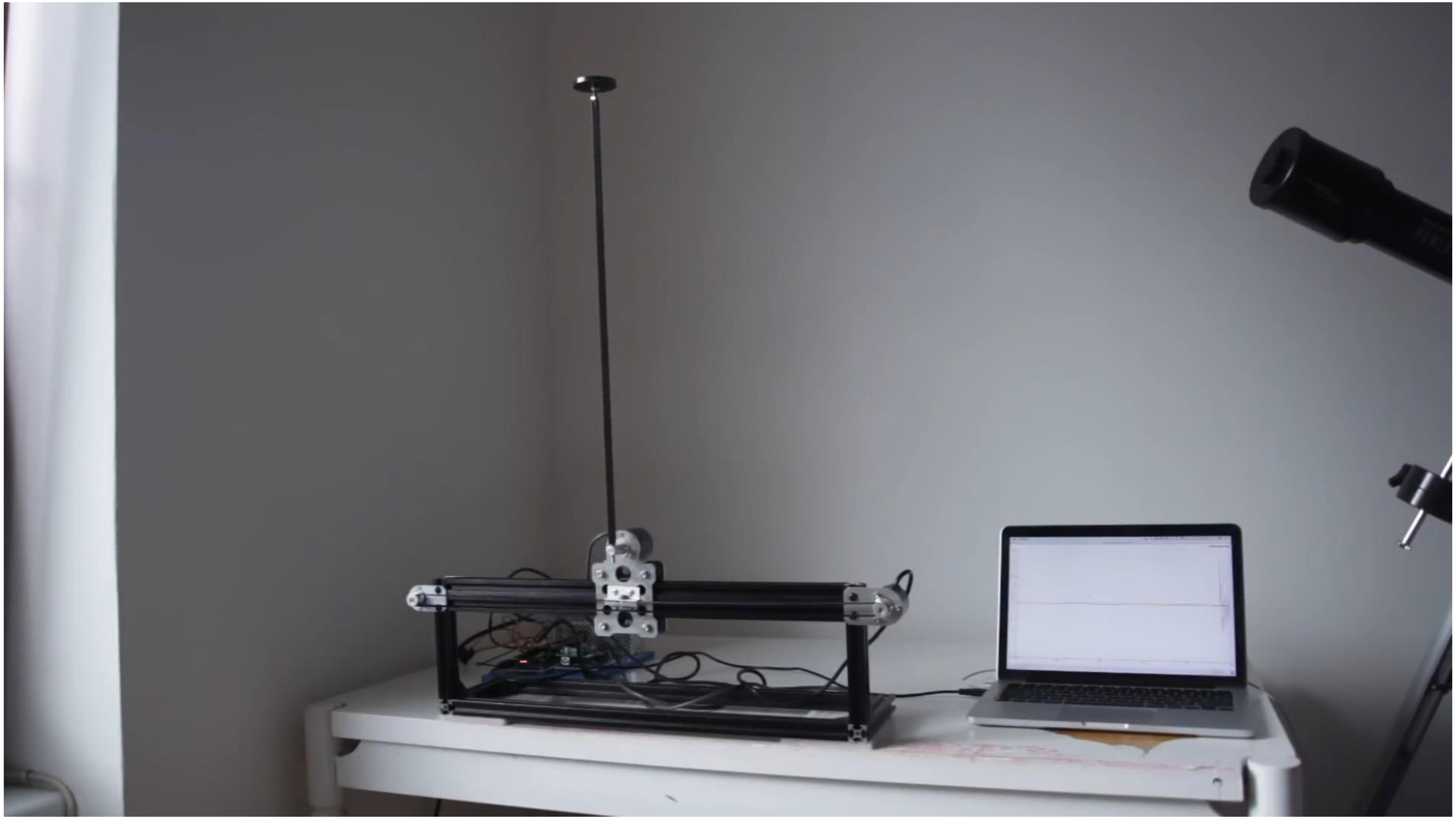
Inverted Pendulum Model (IPM)





Inverted pendulum on a cart

Inverted Pendulum Model (IPM)



Inverted pendulum on a cart https://www.youtube.com/watch?v=nOSTzpA0nGk

Inverted Pendulum Model (IPM)

- Step Plan with IPM

Generalized Biped Walking Control

Stelian Coros Philippe Beaudoin Michiel van de Panne*

University of British Columbia

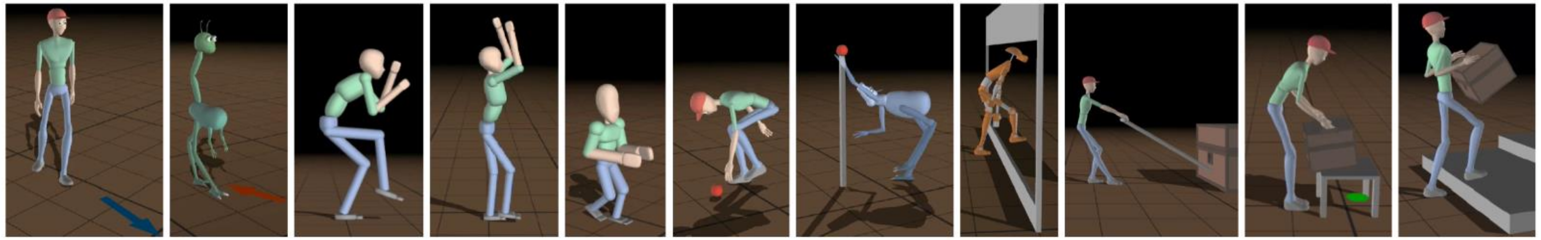
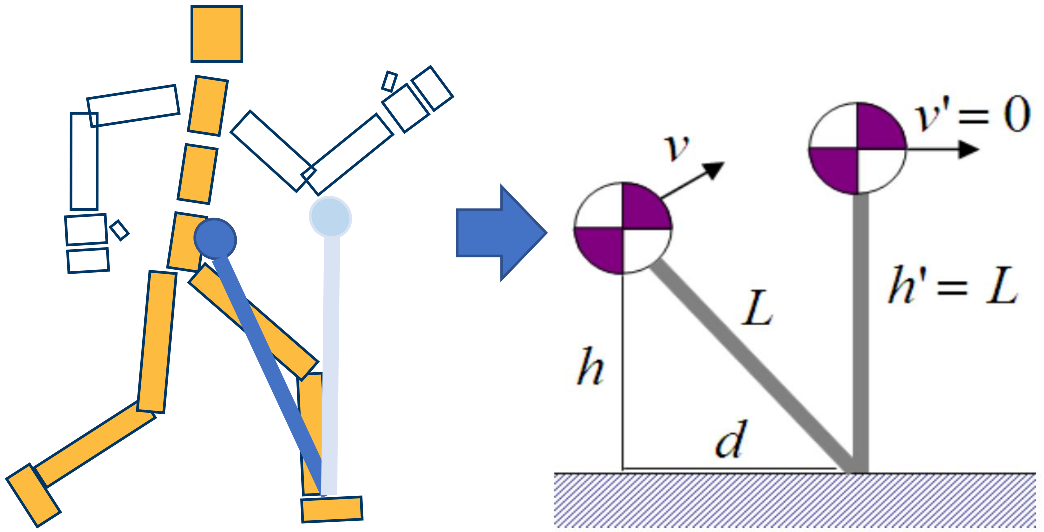


Figure 1: *Real-time physics-based simulation of walking. The method provides robust control across a range of gaits, styles, characters, and skills. Motions are easily authored by novice users.*

[Coros et al. 2010 - **Generalized Biped Walking Control**]

Inverted Pendulum Model (IPM)

- Step Plan with IPM
  - Map CoM of the character and the stance foot as IPM
  - Plan the position of the next foot step so that the mass point rests at the top of the pendulum
  - Create foot trajectory based on the step plan
  - Compute target poses using IK



[Coros et al. 2010 - **Generalized Biped Walking Control**]

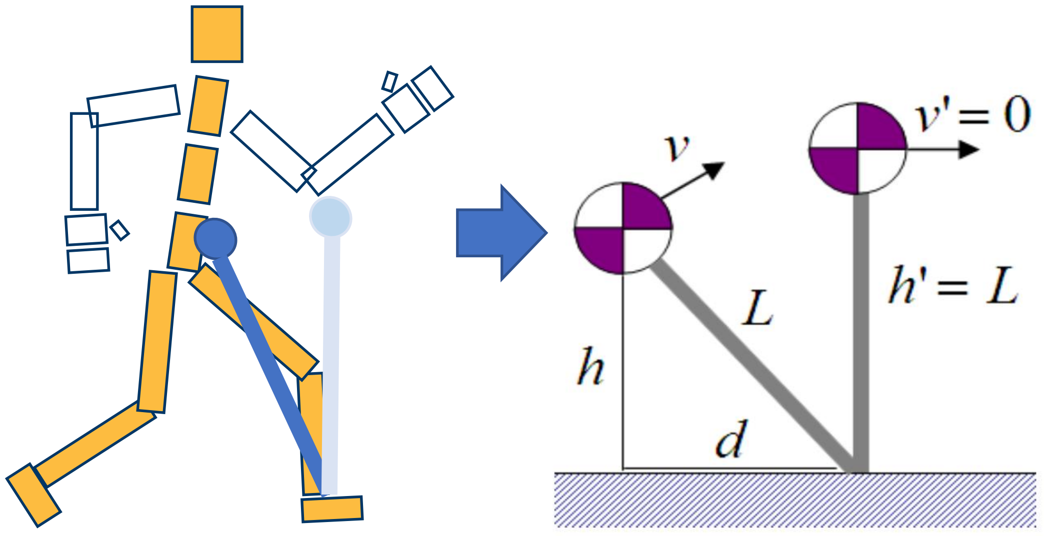
Inverted Pendulum Model (IPM)

- Step Plan with IPM

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv'^2 + mgh'$$

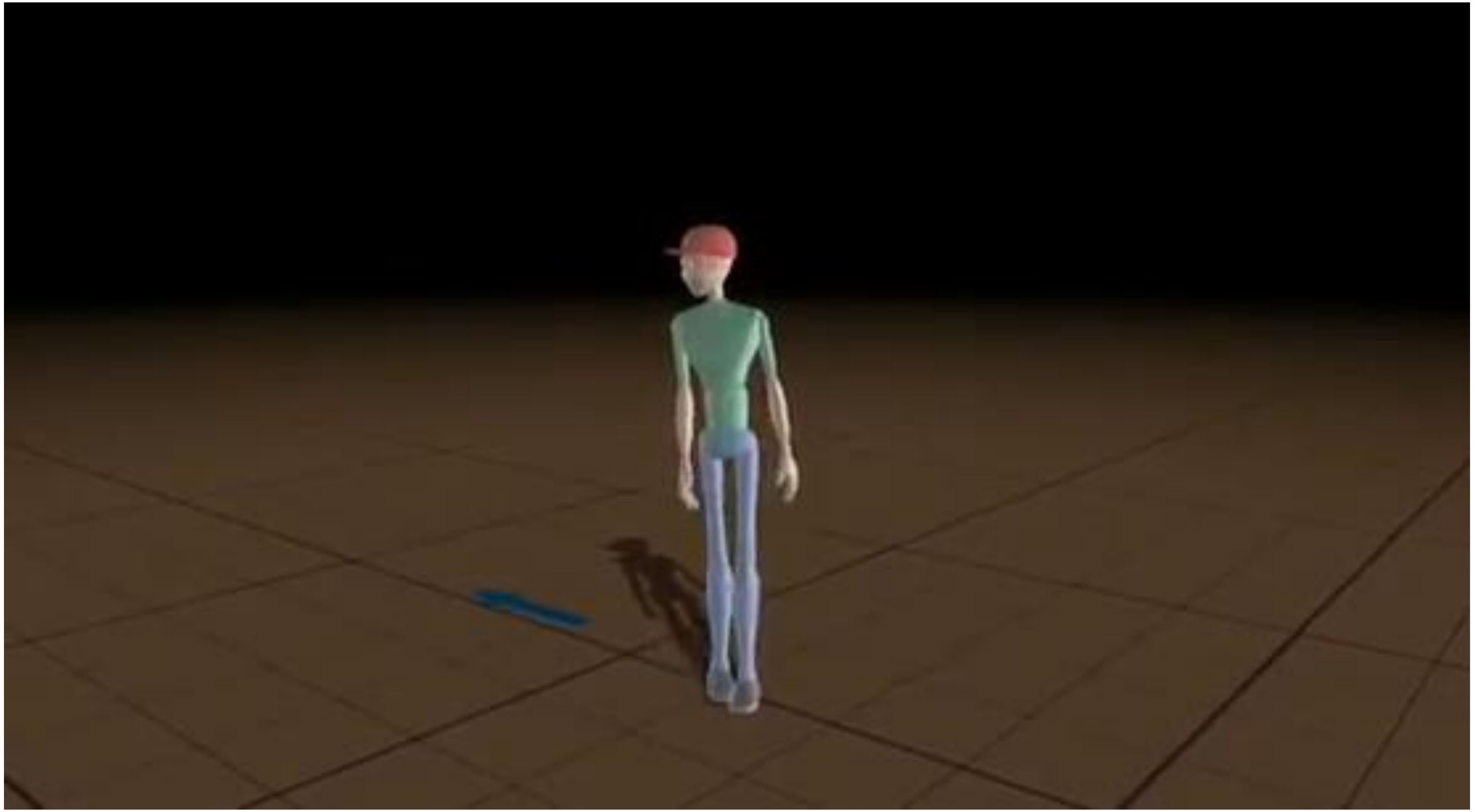
$$v' = 0 \text{ and } h' = L = \sqrt{h^2 + d^2}$$

$$d = v\sqrt{h/g + v^2/(4g^2)}.$$



[Coros et al. 2010 - **Generalized Biped Walking Control**]

Generalized walking control



[Coros et al. 2010]

SIMBICON

- SIMBICON (SIMple BIped Locomotion CONtrol)
  - Yin et al. 2007

SIMBICON: Simple Biped Locomotion Control

KangKang Yin Kevin Loken Michiel van de Panne*

University of British Columbia

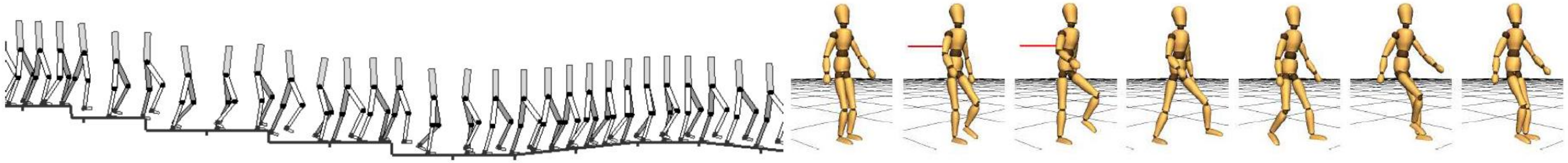
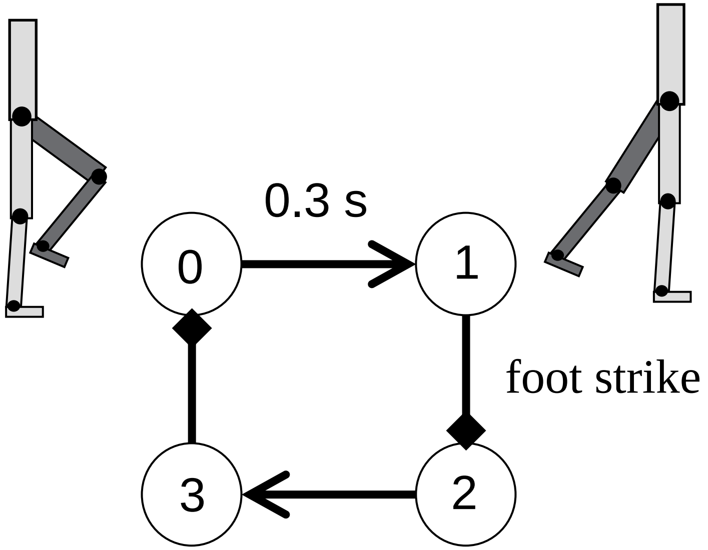
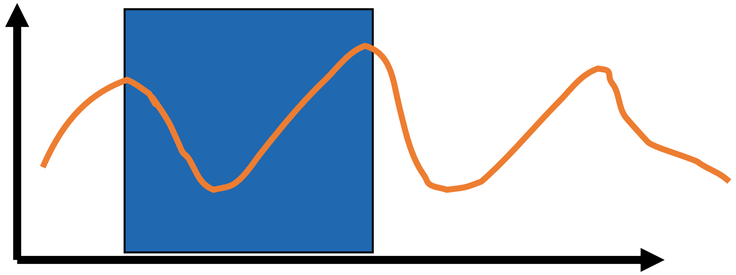


Figure 1: *Real-time physics-based character simulation with our framework. (a) A single controller for a planar biped responds to unanticipated changes in terrain. (b) A walk controller reconstructed from motion capture data responds to a 350N, 0.2s diagonal push to the torso.*

SIMBICON

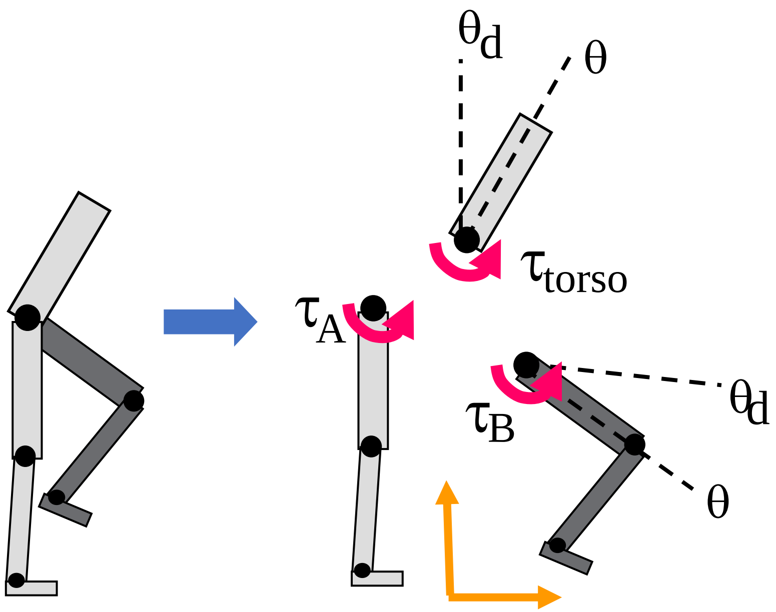
- Step 1: develop a cyclical base motion
  - PD controllers track target angles
  - FSM (Finite State Machine) or mocap





SIMBICON

- Step 2:
  - control torso and swing-hip wrt world frame

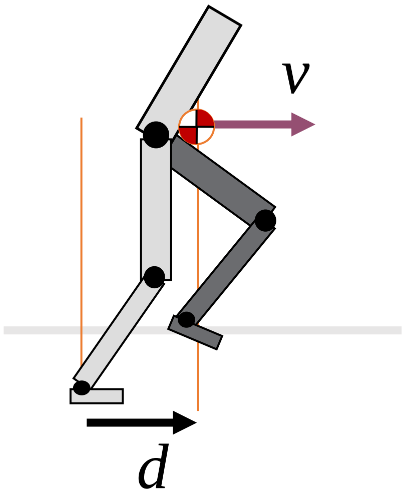


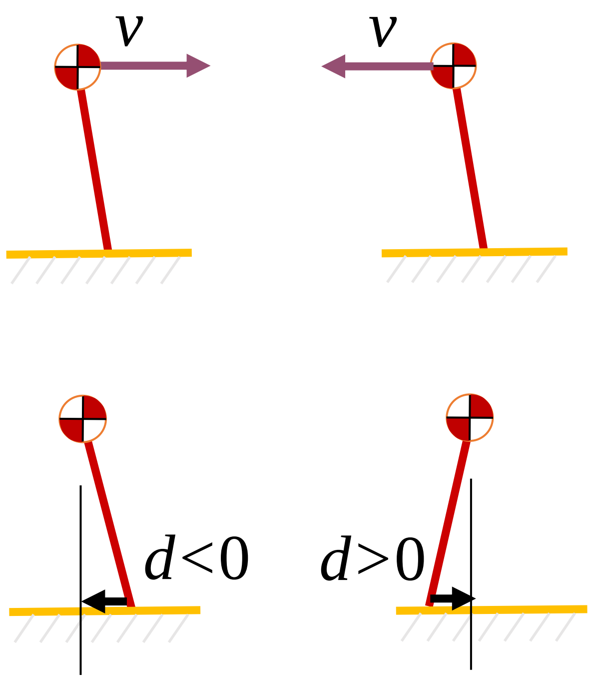
$$\tau_A = -\tau_{\mathrm{torso}} - \tau_B$$

Newton's third law

SIMBICON

- Step 3: COM feedback



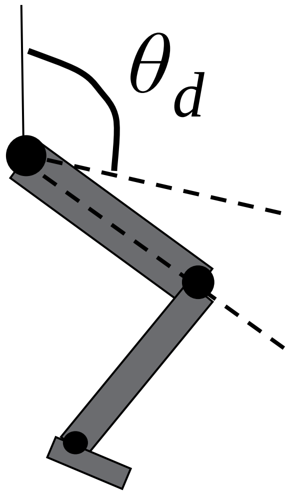


COM velocity matters

COM position matters

SIMBICON

- Step 3: COM feedback



base controller

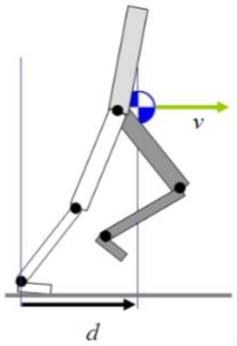
continuous feedback

$$\theta_d = \theta_{d0} + c_d d + c_v v$$

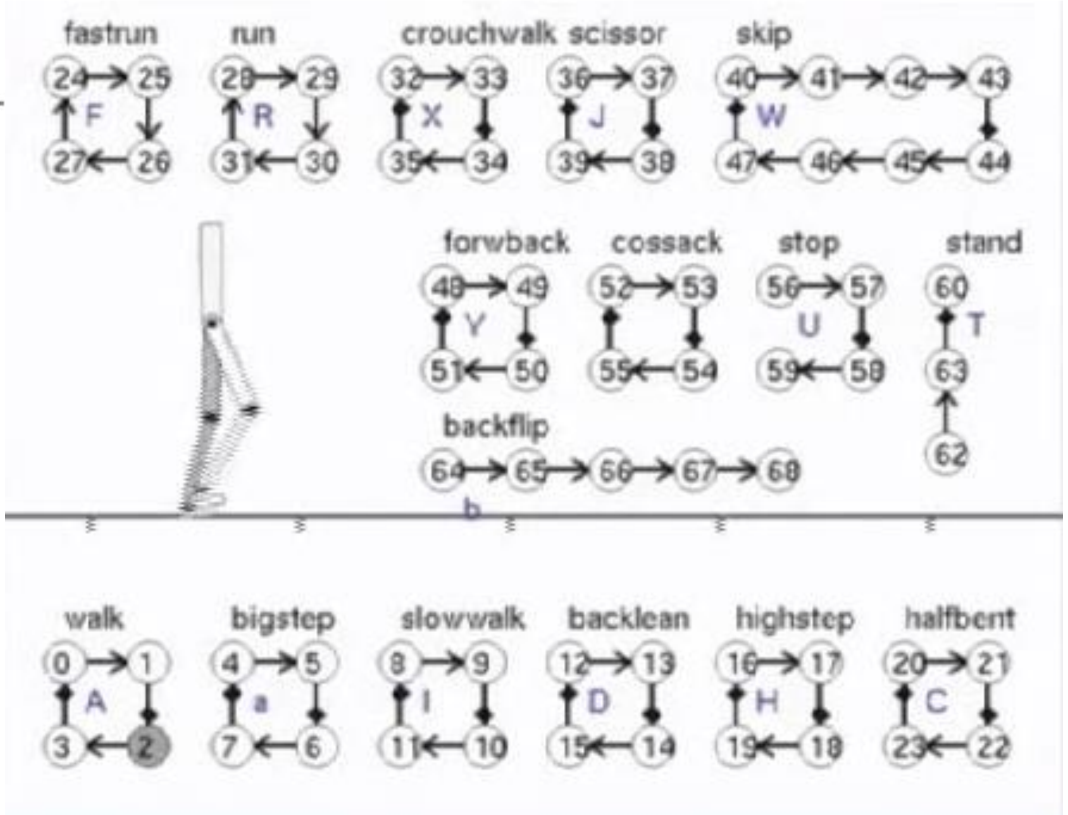
COM position

COM velocity

SIMBICON



Linear Feedback Control + FSM



[Yin et al. 2007, SIMBICON]

Outline

- Walking and Dynamic Balance

- Simplified Models
  - ZMP (Zero-Moment Point)
  - Inverted Pendulum
  - SIMBICON

- How to generalize to other motion?

Questions?