Lecture 10

Controlling Characters

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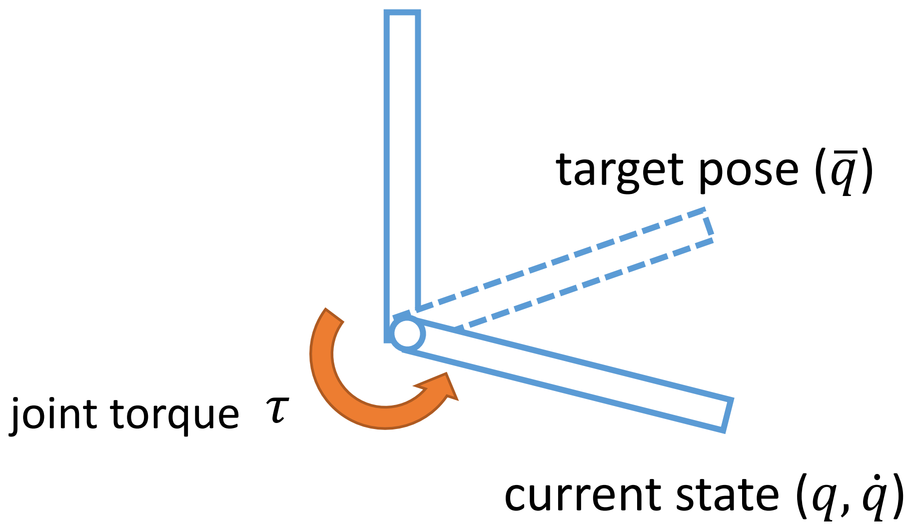
GAMES105 课程交流

VCL @ PKU

Outline

- More about PD (Proportional-Derivative) control
  - Stable PD control
- Feedforward Motion Control
  - Trajectory optimization
- Feedback Motion Control
  - Static balance

PD Control for Characters



target pose

$$\tau = k_p(\bar{q} - q) - k_d\dot{q}$$

proportional control

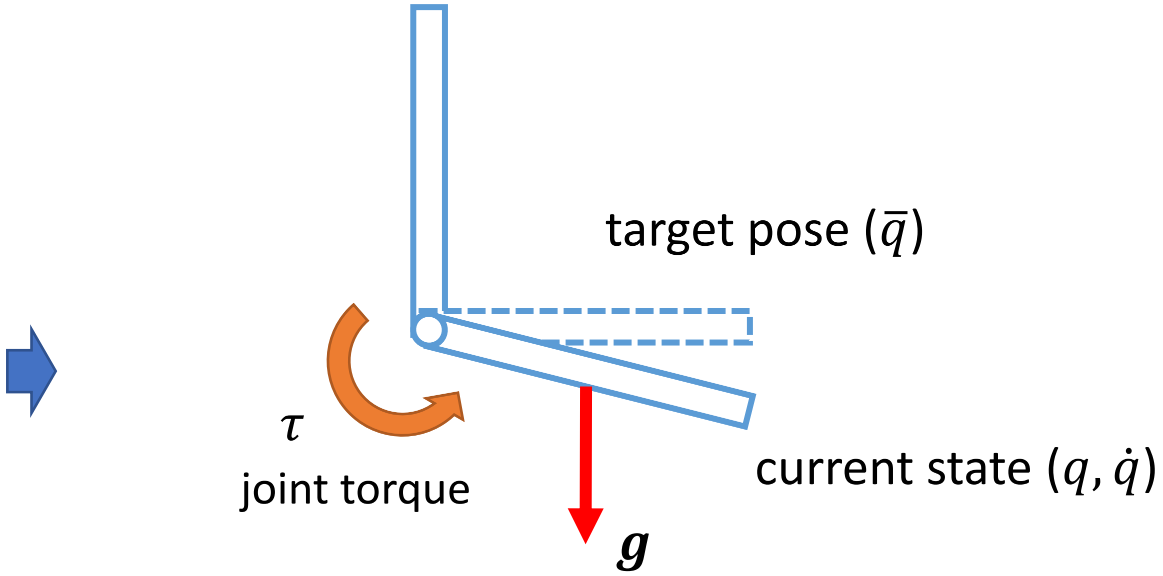
derivative control

Problems with PD Control

PD control computes torques based on errors

- Steady state error

This arm never reaches the target angle under gravity

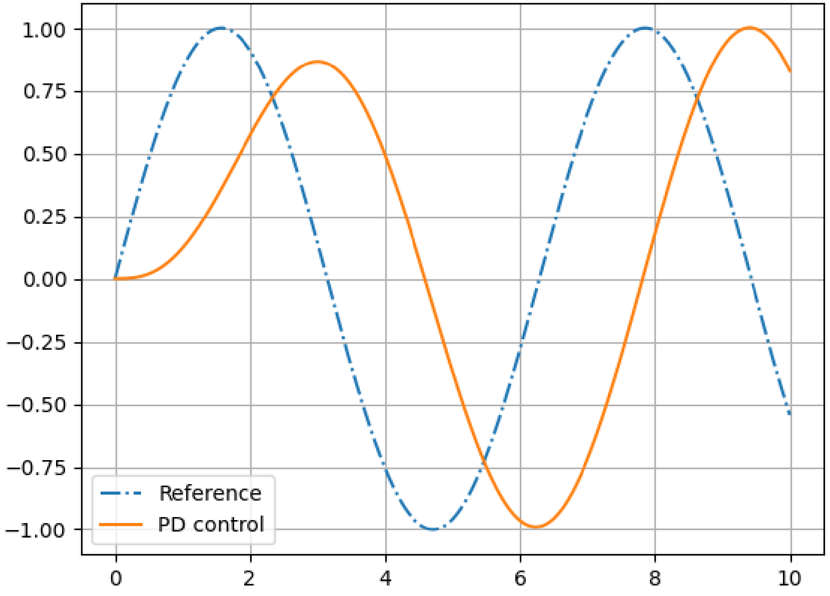


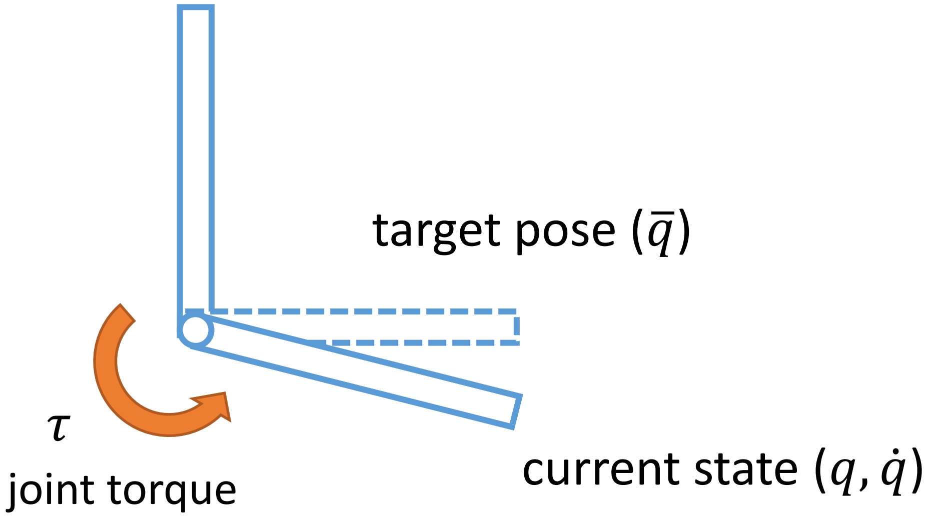
$$\tau = k_p(\bar{q} - q) - k_d\dot{q}$$

Problems with PD Control

PD control computes torques based on errors

- Steady state error
- Motion falls behind the reference





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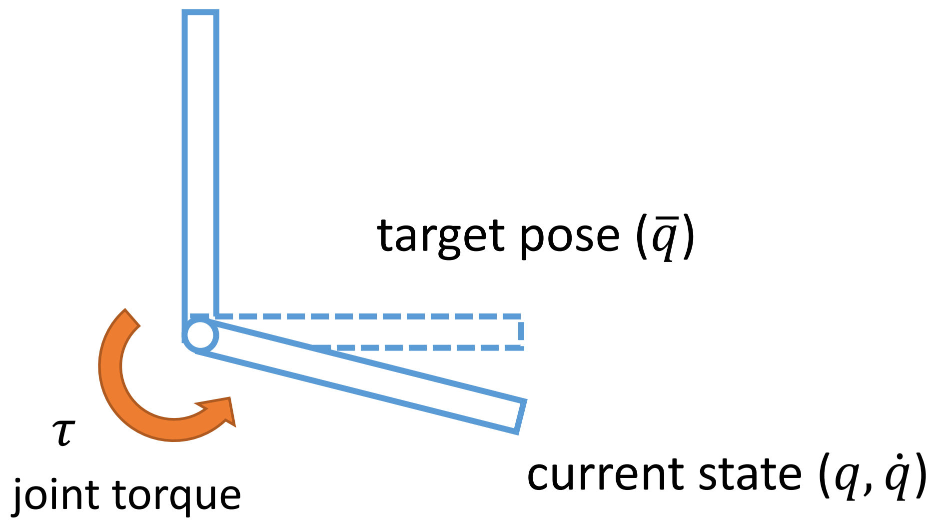
High-gain (k_p) control is more precise…



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Problems with PD Control

PD control computes torques based on errors

- Steady state error
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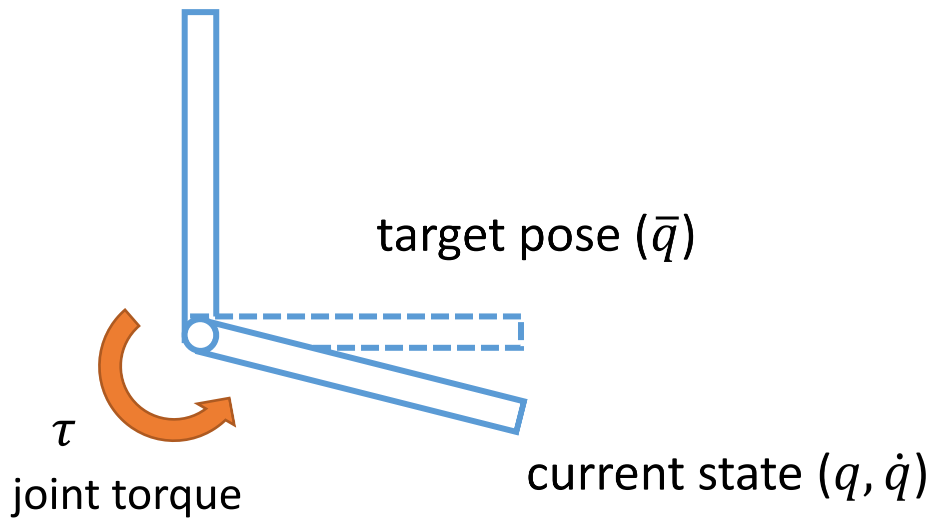
High-gain (k_p) control is more precise but less stable…



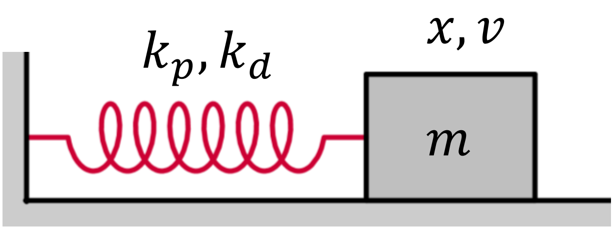
$$\tau = k_p(\bar{q} - q) - k_d\dot{q}$$

Stability of PD Control

Semi-implicit Euler Integration

$$v_{n+1} = v_n + h\frac{f}{m}$$

$$x_{n+1} = x_n + hv_{n+1}$$



$$f = -k_p x - k_d v$$

h: simulation time step

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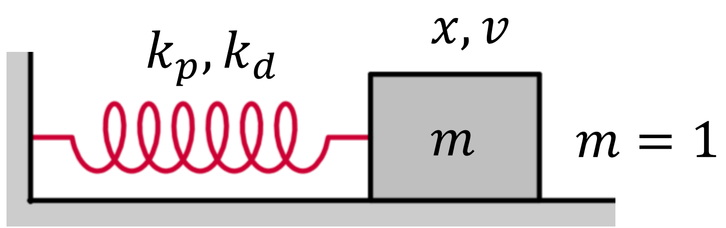

$$f = -k_p x - k_d v$$

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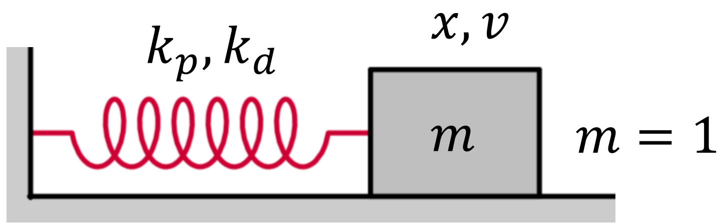

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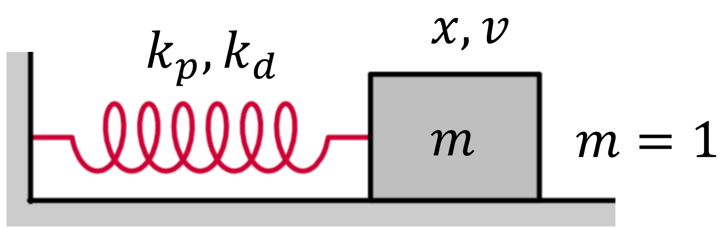

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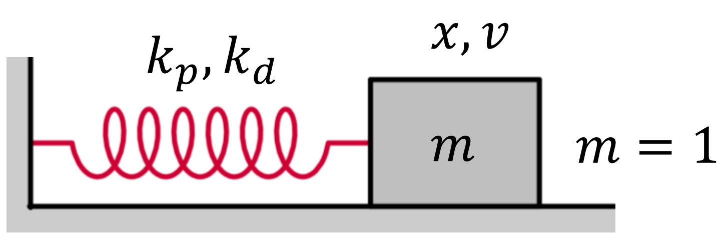

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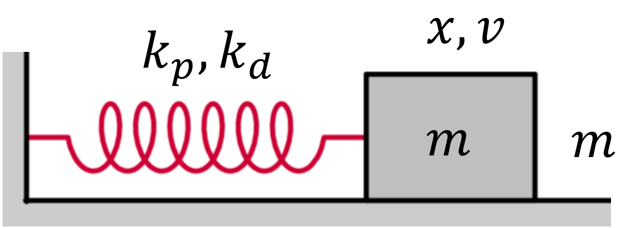
Stability of PD Control

$$\boldsymbol{s}_{n+1} = A\boldsymbol{s}_n \quad \Rightarrow \quad \boldsymbol{s}_{n+1} = A^n\boldsymbol{s}_1$$

Assume A has two eigenvalues λ_1, λ_2 and two eigen vectors $\boldsymbol{v}_1, \boldsymbol{v}_2$. Note $\lambda_i \in \mathbb{C}$

$$A = P\begin{bmatrix}\lambda_1 & \\ & \lambda_2\end{bmatrix}P^{-1} \qquad P = \begin{bmatrix}\vdots & \vdots \\ \boldsymbol{v}_1 & \boldsymbol{v}_2 \\ \vdots & \vdots\end{bmatrix}$$



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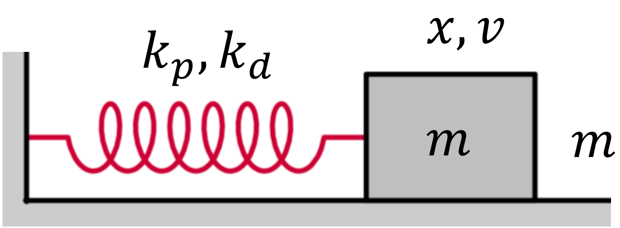

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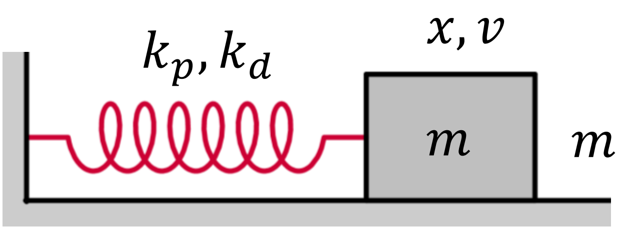

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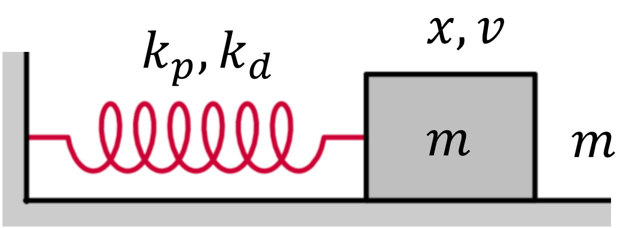

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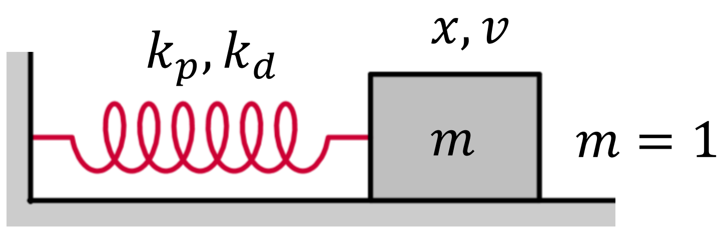

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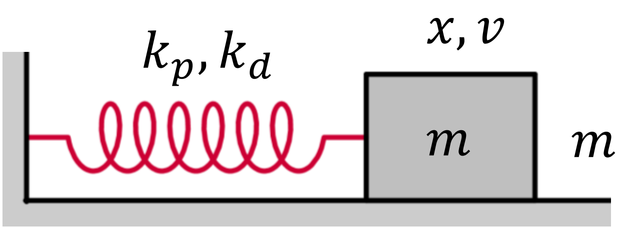
$\lambda_1, \lambda_2 \in \mathbb{C}$ are eigenvalues of A

if $|\lambda_1| > 1$ $\Rightarrow$ $\lim_{n\to\infty} \|\boldsymbol{z}_n\| \to \infty$

The system is unstable!



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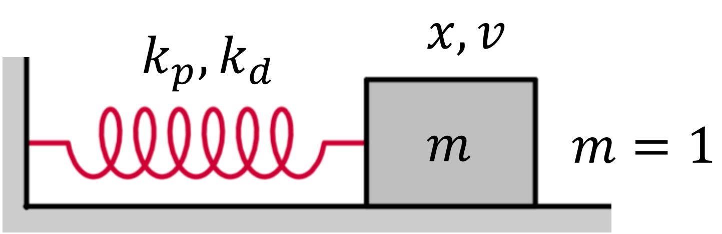
The system is unstable!

Condition of stability: $|\lambda_i| \leq 1$ for all λ_i



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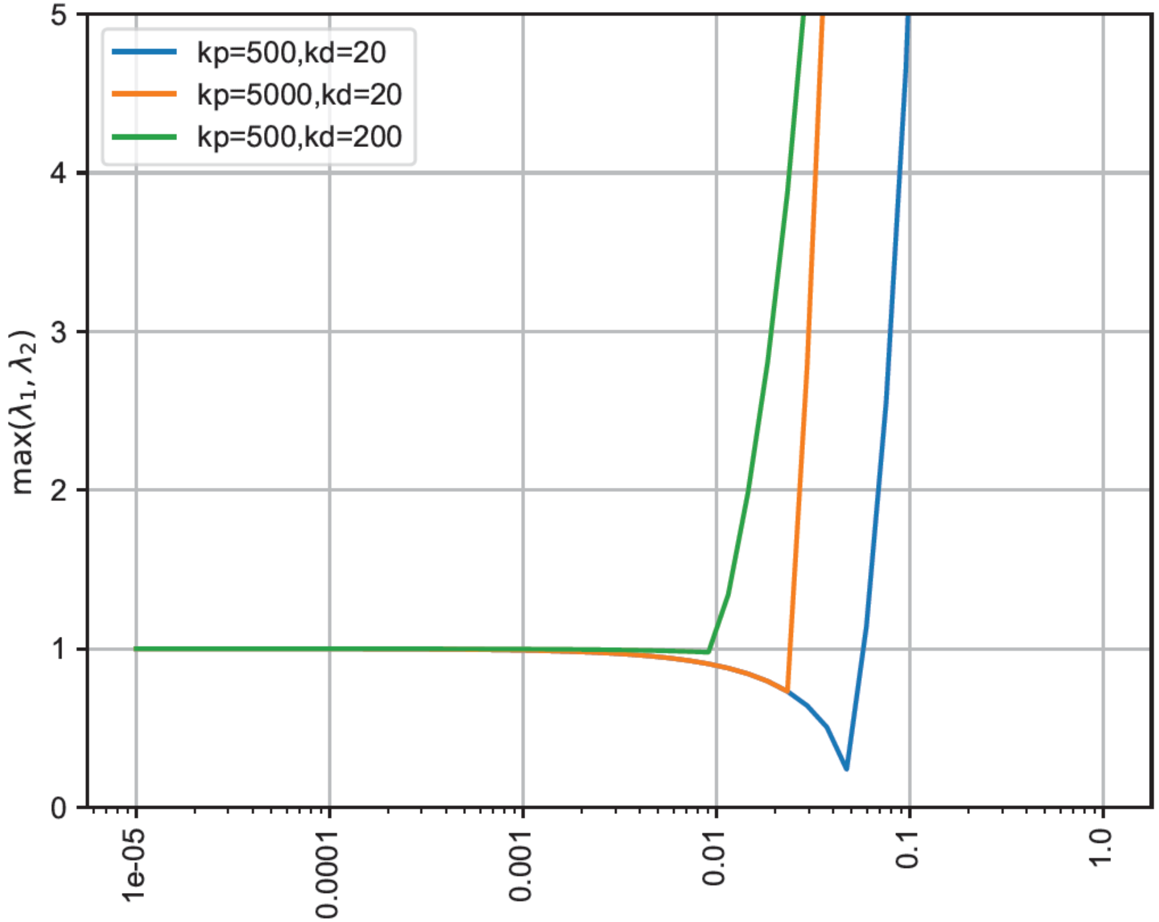

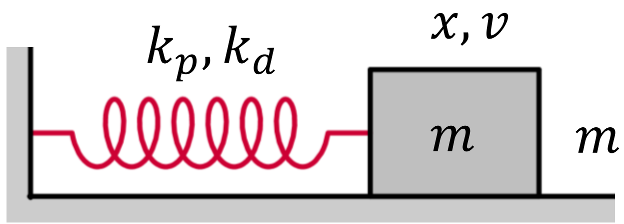

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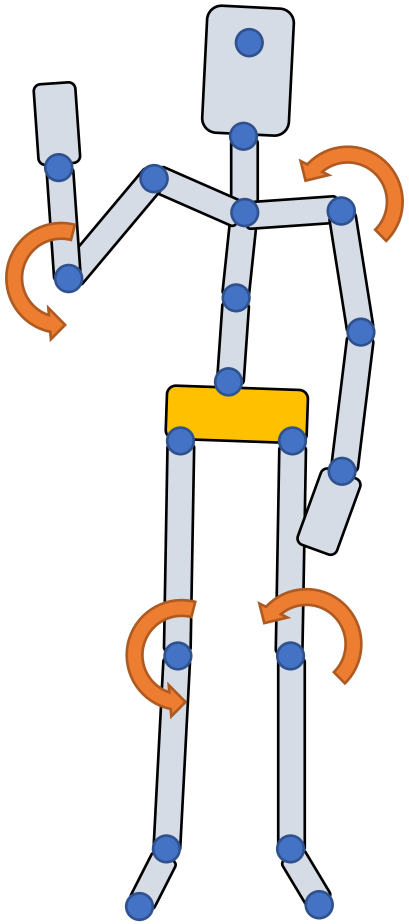
PD Control for Characters



$$\tau = k_p(\bar{q} - q) - k_d\dot{q}$$

- Determining gain and damping coefficients can be difficult…
  - A typical setting $k_p = 200$, $k_d = 20$ for a 50kg character
  - Light body requires smaller gains
  - Dynamic motions need larger gains
- High-gain/high-damping control can be unstable, so small times is necessary
  - $h = 0.5{\sim}1\text{ms}$ is often used, or $1000{\sim}2000\text{Hz}$
  - Higher gain/damping requires smaller time step

A More Stable PD Control

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$$v_{n+1} = v_n + h(-k_p x_n - k_d v_n)$$

$$x_{n+1} = x_n + h v_{n+1}$$

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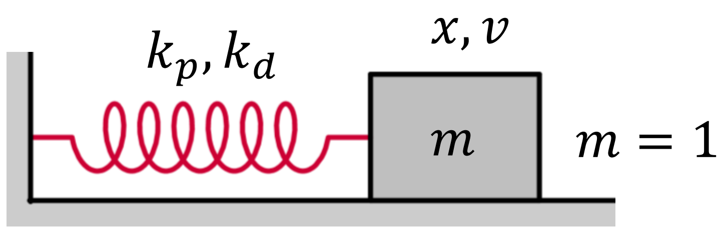

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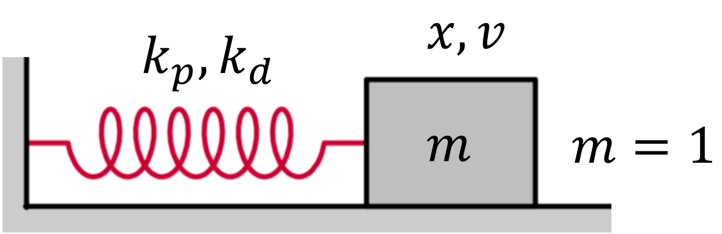
$$\begin{bmatrix} v_{n+1} \\ x_{n+1} \end{bmatrix} = \frac{1}{1 + hk_d} \begin{bmatrix} 1 & -k_p h \\ h & 1 + k_d h - k_p h^2 \end{bmatrix} \begin{bmatrix} v_n \\ x_n \end{bmatrix}$$



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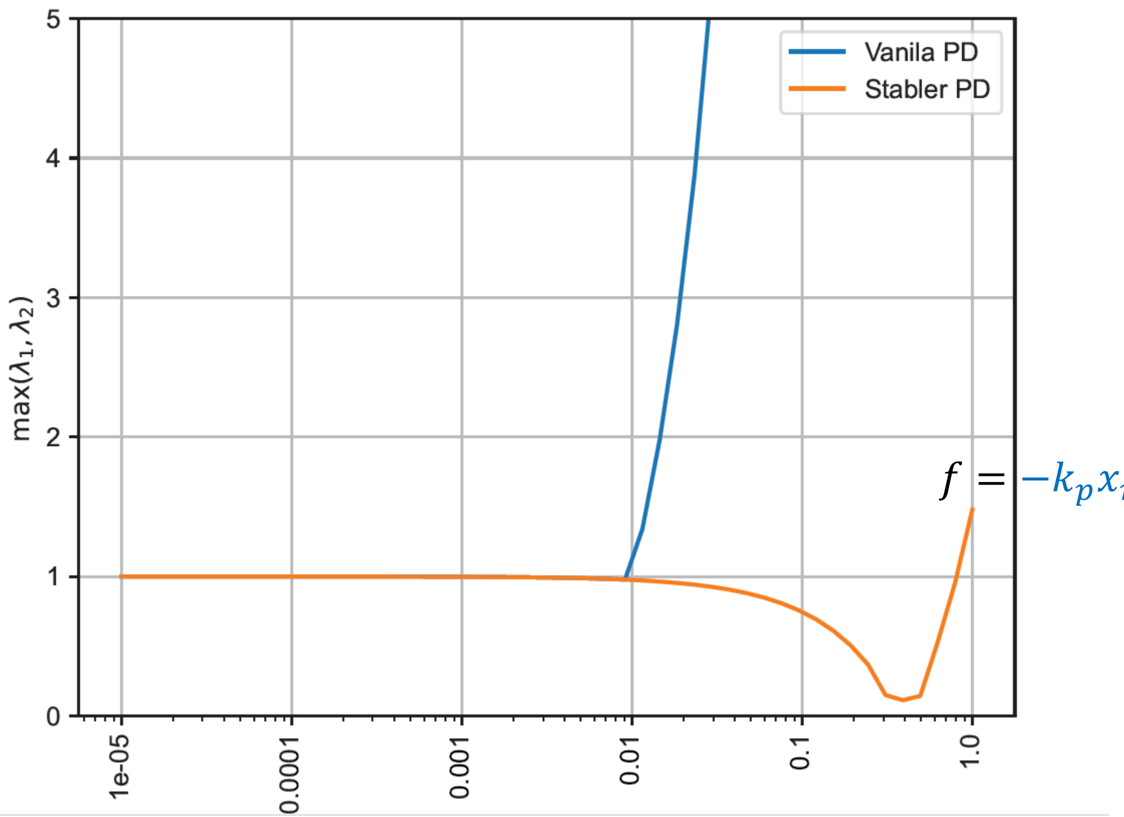

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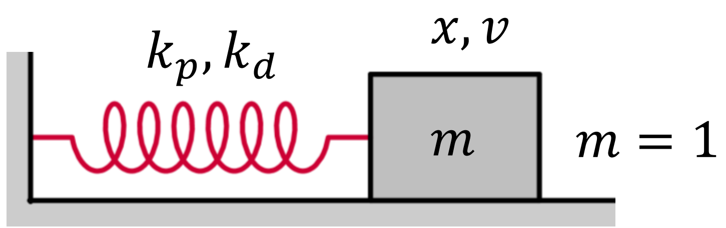

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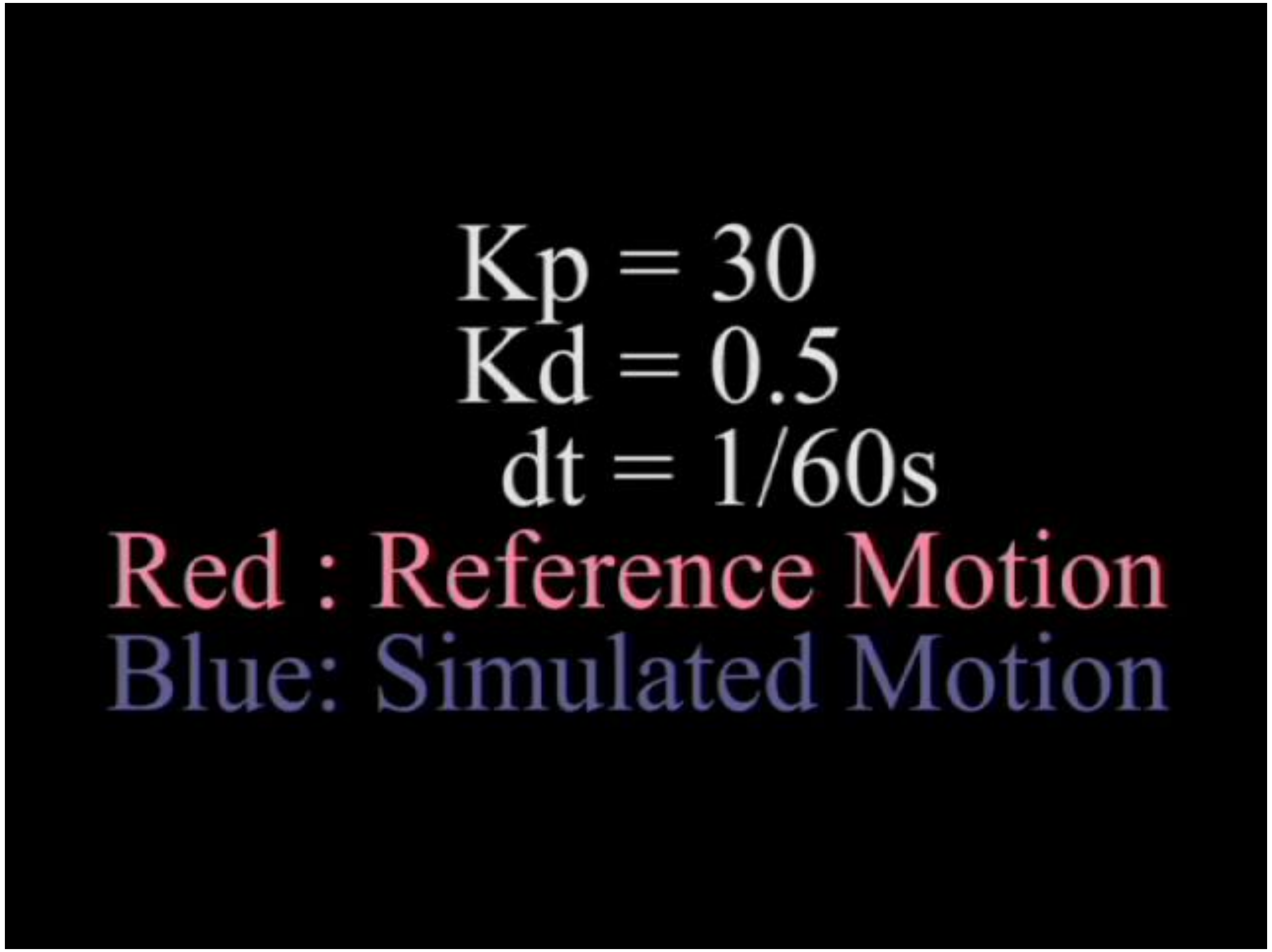
Stable PD Control

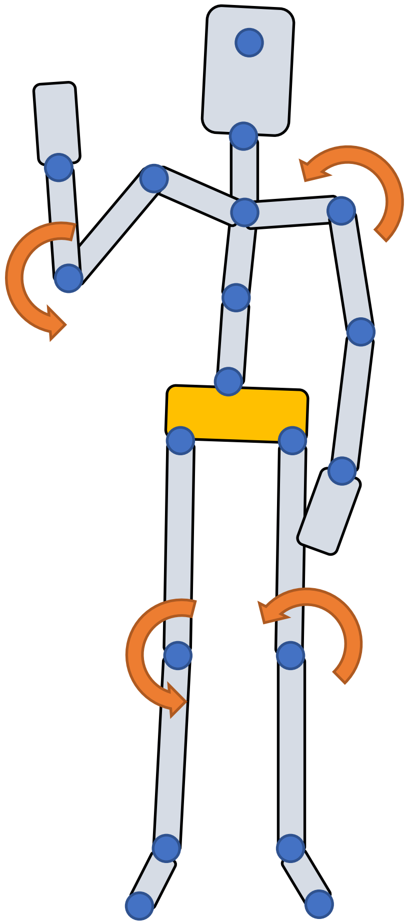


$$\tau_{\text{int}} = -\mathbf{K}_p(\mathbf{q}^n + \dot{\mathbf{q}}^n \Delta t - \bar{\mathbf{q}}^{n+1}) - \mathbf{K}_d(\dot{\mathbf{q}}^n + \ddot{\mathbf{q}}^n \Delta t)$$

Stable Proportional-Derivative Controllers

Jie Tan* Karen Liu† Greg Turk‡
Georgia Institute of Technology

PD Control for Characters



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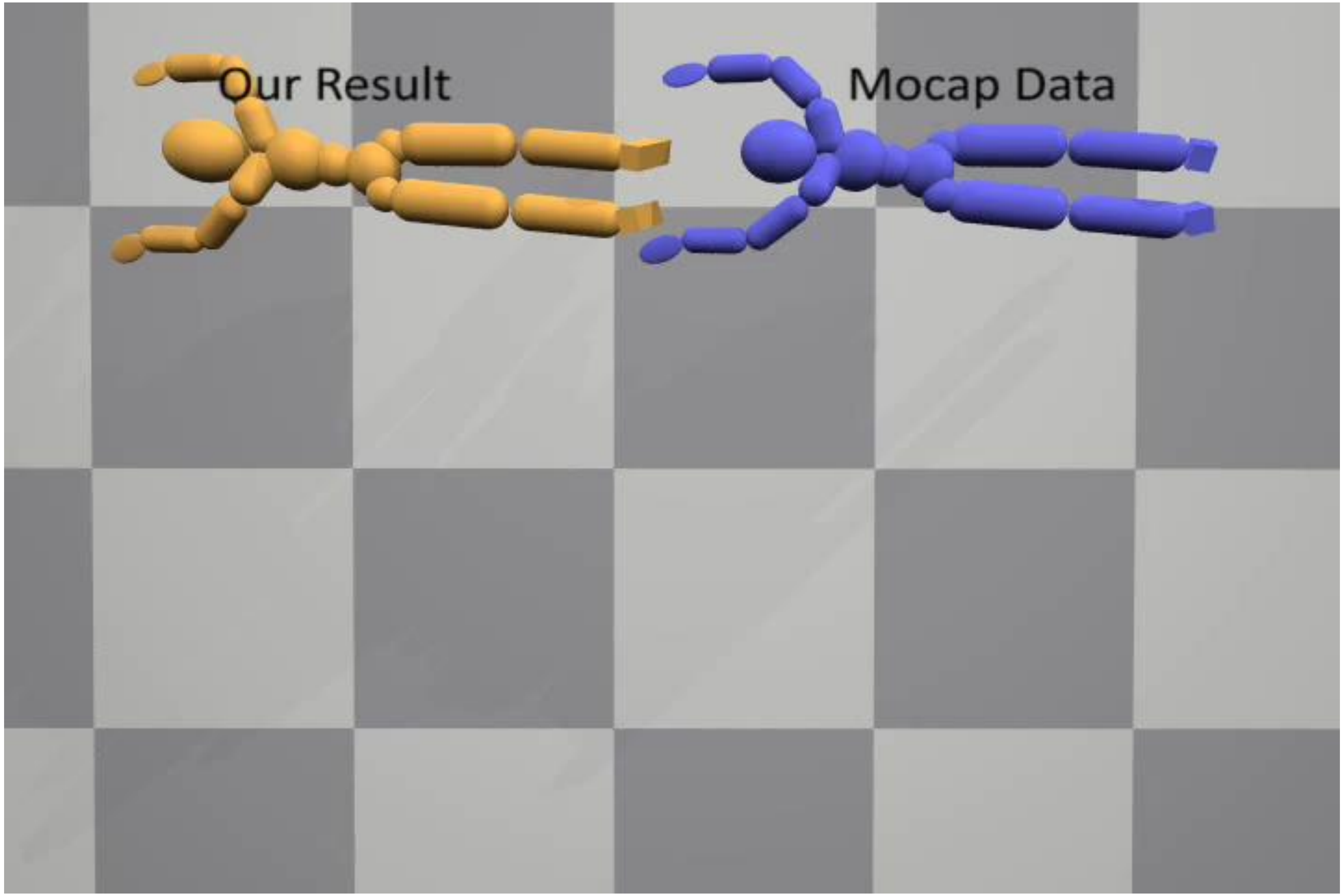
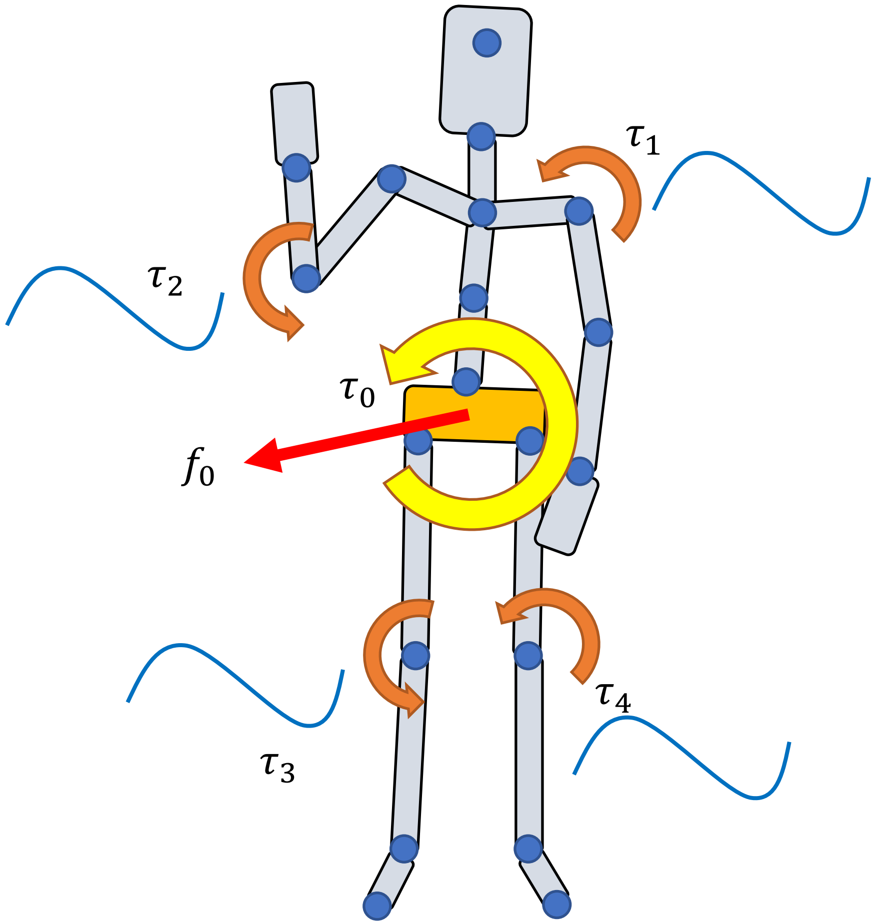
  - $h = 0.5\sim1\mathrm{ms}$ is often used, or $1000\sim2000\mathrm{Hz}$
  - $h = 1/120\mathrm{s}\sim1/60\mathrm{s}$, or 120Hz/60Hz with Stable PD
  - Higher gain/damping requires smaller time step

Tracking Mocap



[SAMCON – Liu et al 2010]

Tracking Mocap with Root Forces/Torques



τ_j: joint torques

Apply τ_j to “child” body

Apply $-\tau_j$ to “parent” body

All forces/torques sum up to zero

f_0, τ_0: root force / torque

Apply f_0 to the root body

Apply τ_0 to the root body

Non-zero net force/torque on the character!

Physically Plausible Animation

Party Animals

Totally Accurate Battle Simulator
https://www.youtube.com/watch?v=WFKGWfdG3bU

Trajectory Optimization

Find the trajectories:

Simulation trajectory: $s_0, s_1, \dots, s_T$

Control trajectory: $a_0, a_1, \dots, a_{T-1}$

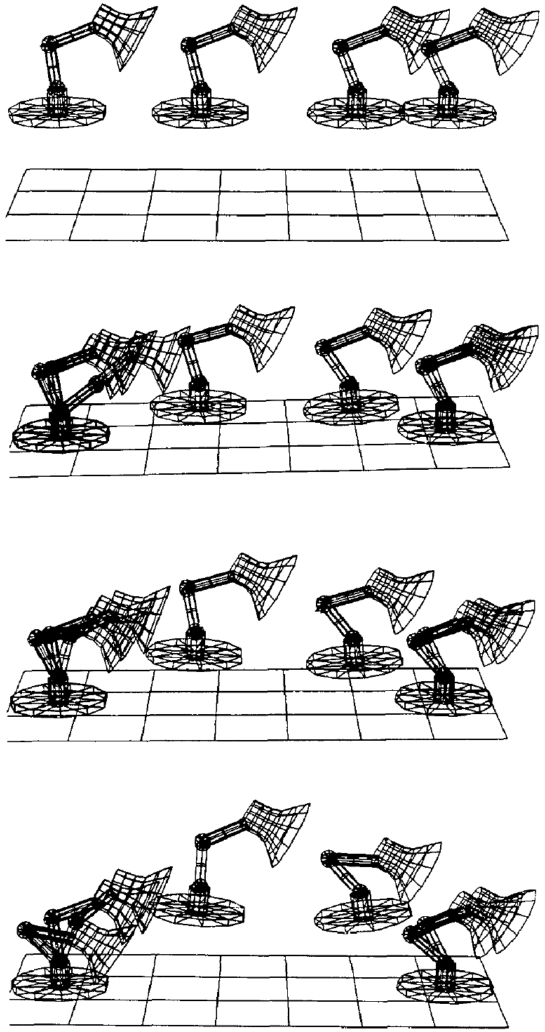
that minimize the objective function

$$\min_{\{s_t, a_t\}} f(s_T) + \sum_{t=0}^{T-1} f(s_t, a_t)$$

and satisfy the constraints:

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + J^T\lambda \qquad \text{Equations of motion}$$

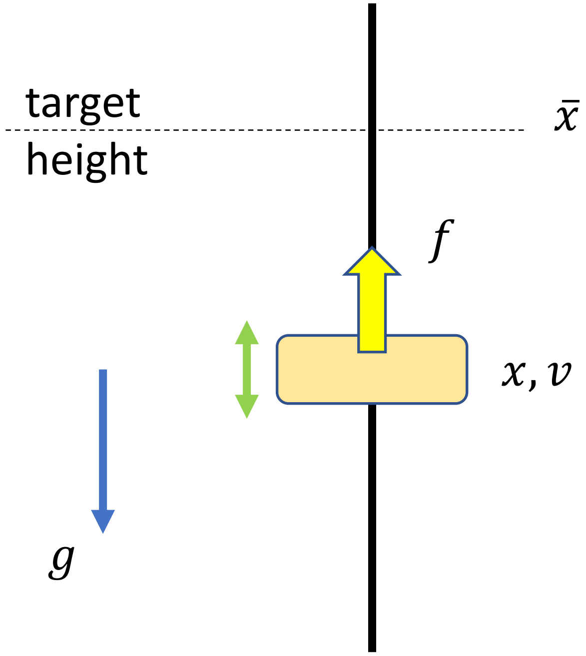
$$g(\boldsymbol{x}, \boldsymbol{v}) \geq 0 \qquad \text{constraints}$$



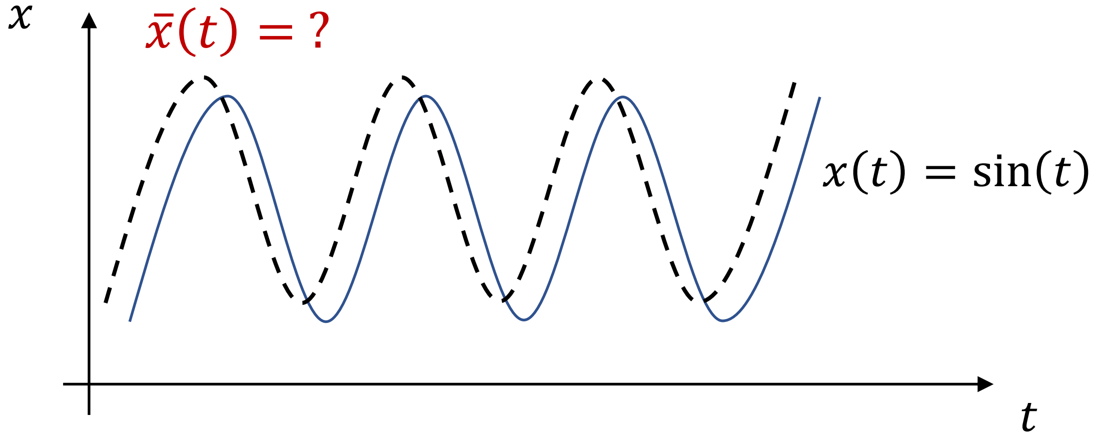
[Witkin and Kass 1988 – Spacetime constraints]

A very simple example

$$f = k_p(\bar{x} - x) - k_d v$$



Compute a target trajectory $\bar{x}(t)$
such that the simulated trajectory $x(t)$
is a sine curve.



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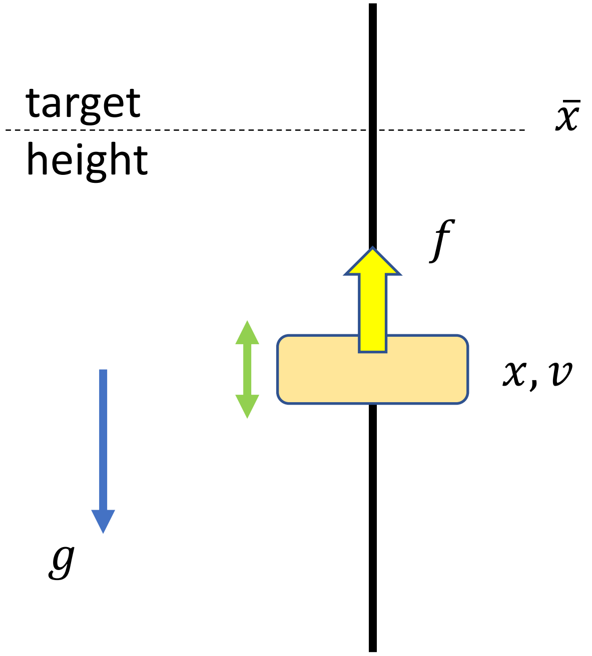

$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} (\sin(t_n) - x_n)^2$$

$$s.t. \quad v_{n+1} = v_n + h\big(k_p(\bar{x}_n - x_n) - k_d v_n\big)$$

$$x_{n+1} = x_n + h v_{n+1}$$

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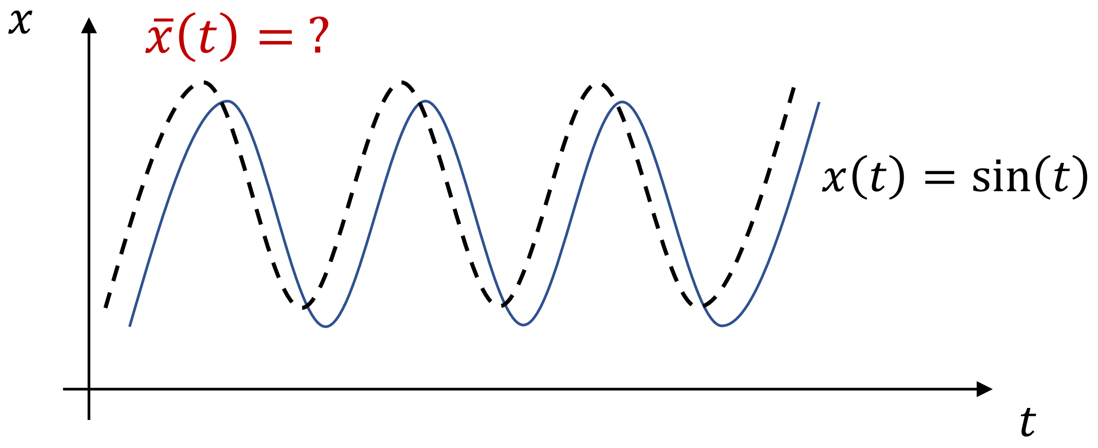

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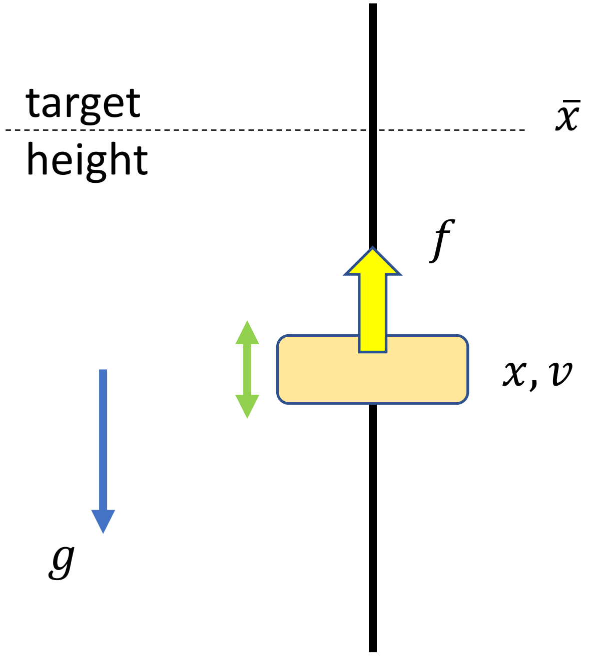

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such that the simulated trajectory $x(t)$
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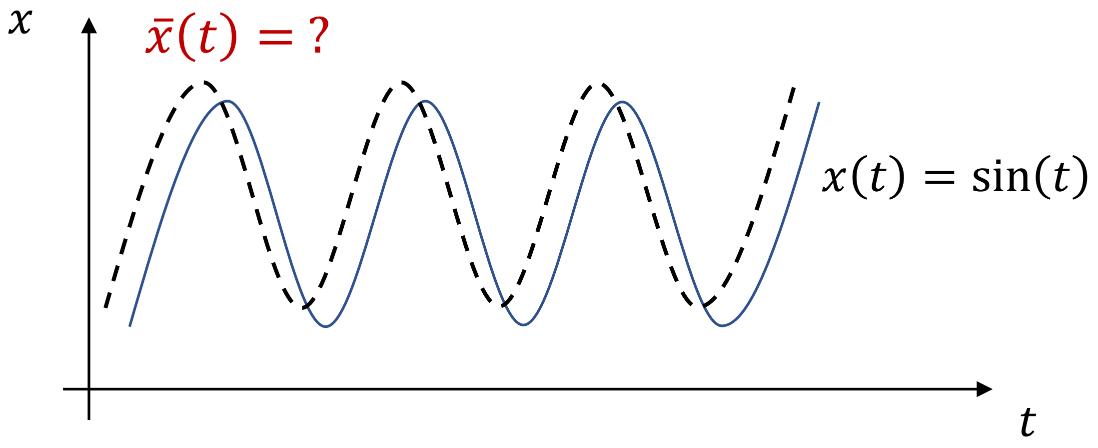
$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} (\sin(t_n) - x_n)^2 + \sum_{n=0}^{N} \bar{x}_n^2$$

$$s.t. \quad v_{n+1} = v_n + h\big(k_p(\bar{x}_n - x_n) - k_d v_n\big)$$

$$x_{n+1} = x_n + h v_{n+1}$$



A very simple example

Hard constraints:

$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} (\sin(t_n) - x_n)^2 + w_{\bar{x}} \sum_{n=0}^{N} \bar{x}_n^2$$

$$s.t. \quad v_{n+1} = v_n + h\big(k_p(\bar{x}_n - x_n) - k_d v_n\big)$$

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Soft constraints:

$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} (\sin(t_n) - x_n)^2 + w_{\bar{x}} \sum_{n=0}^{N} \bar{x}_n^2$$

$$+ w_v \sum_{n=0}^{N} \Big(v_{n+1} - v_n - h\big(k_p(\bar{x}_n - x_n) - k_d v_n\big)\Big)^2$$

$$+ w_x \sum_{n=0}^{N} (x_{n+1} - x_n - h v_{n+1})^2$$

A very simple example

Collocation methods:

$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} \left(\sin(t_n;\theta) - x(t_n;\theta)\right)^2 + w_{\bar{x}} \sum_{n=0}^{N} \bar{x}^2(t_n;\theta)$$

$$s.t. \quad v(t_{n+1};\theta) = v(t_n) + hk_p\left(\bar{x}(t_n;\theta) - x(t_n;\theta)\right) - hk_d v(t_n;\theta)$$

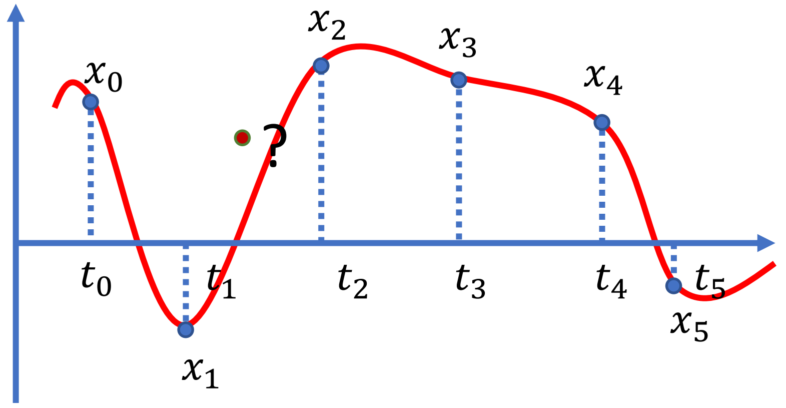
$$x(t_{n+1};\theta) = x(t_n;\theta) + hv(t_{n+1};\theta)$$

Assume the optimization variables $\{x_n, v_n, \bar{x}_n\}$ are values of a set of parametric curves

- typically polynomials or splines

Optimize the parameters of the curves $\boldsymbol{\theta}$ instead

- with smaller number of variables than the original problem



A very simple example

How to solve this optimization problem?

$$\min_{\{x_n, v_n, \bar{x}_n\}} \sum_{n=0}^{N} (\sin(t_n) - x_n)^2 + \sum_{n=0}^{N} \bar{x}_n^2$$

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$$x_{n+1} = x_n + h v_{n+1}$$

A very simple example

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Gradient-based approaches:

- Gradient descent
- Newton's methods
- Quasi-Newton methods
- ……

A very simple example

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Gradient-based approaches:

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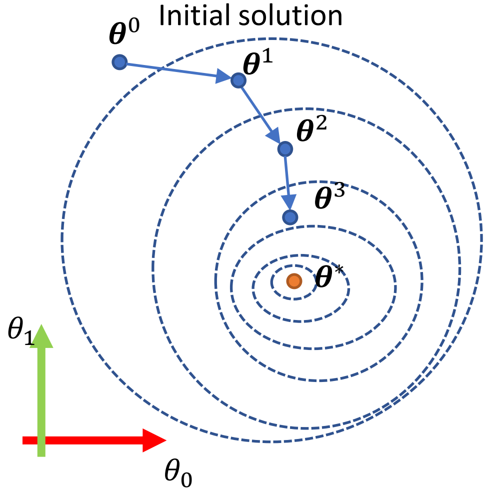
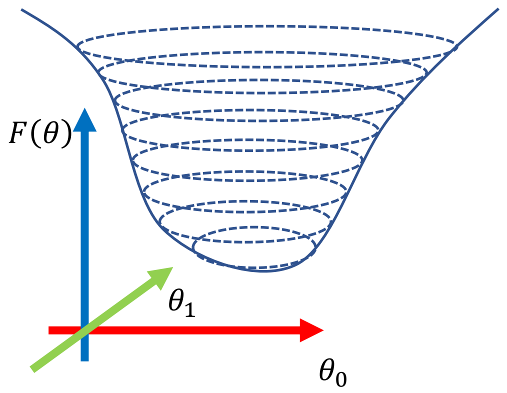
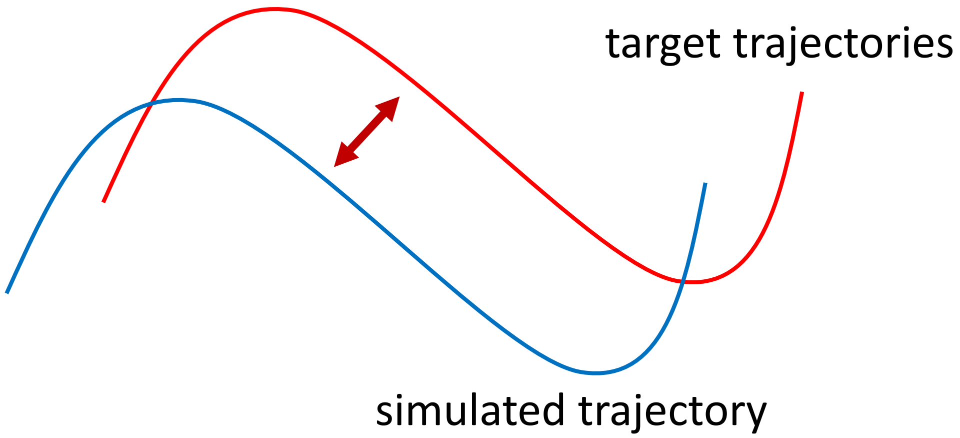
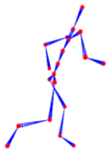




Trajectory Optimization for Tracking Control

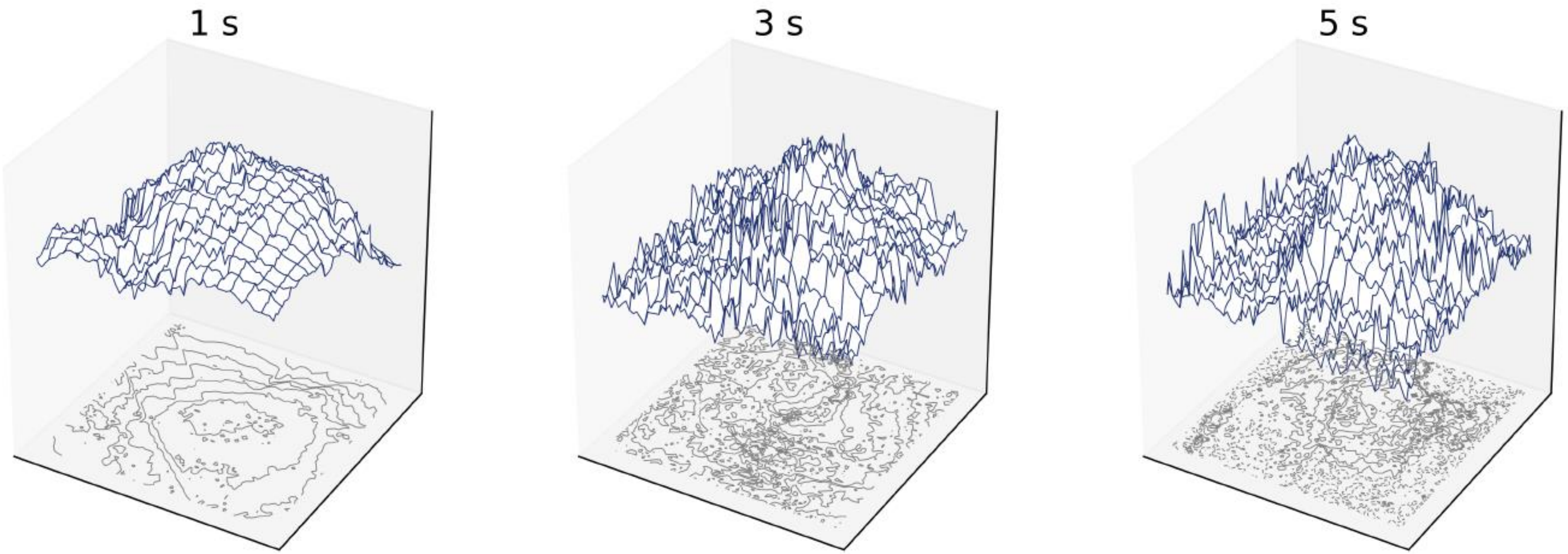


find a target trajectory

minimize ‖  −  ‖ + ⋯

Problem with Gradient-Based Methods

- The optimization problem is usually highly nonlinear, gradients are unreliable
- The system is a black box with unknow dynamics, gradients are not available



Trajectory optimization landscapes of humanoid locomotion

[Hämäläinen et al 2020 - Visualizing Movement Control Optimization Landscapes]

Derivative-Free Optimization

- Iterative methods
  - Goal: find the variables $\boldsymbol{x}$ that optimize $f(\boldsymbol{x})$
  - Determining an initial guess of $\boldsymbol{x}$
  - Repeat:
    - Propose a set of candidate variables $\{\boldsymbol{x}_i\}$ according to $\boldsymbol{x}$
    - Evaluate the objective function $f_i = f(\boldsymbol{x}_i)$
    - Update the estimation for $\boldsymbol{x}$

Derivative-Free Optimization

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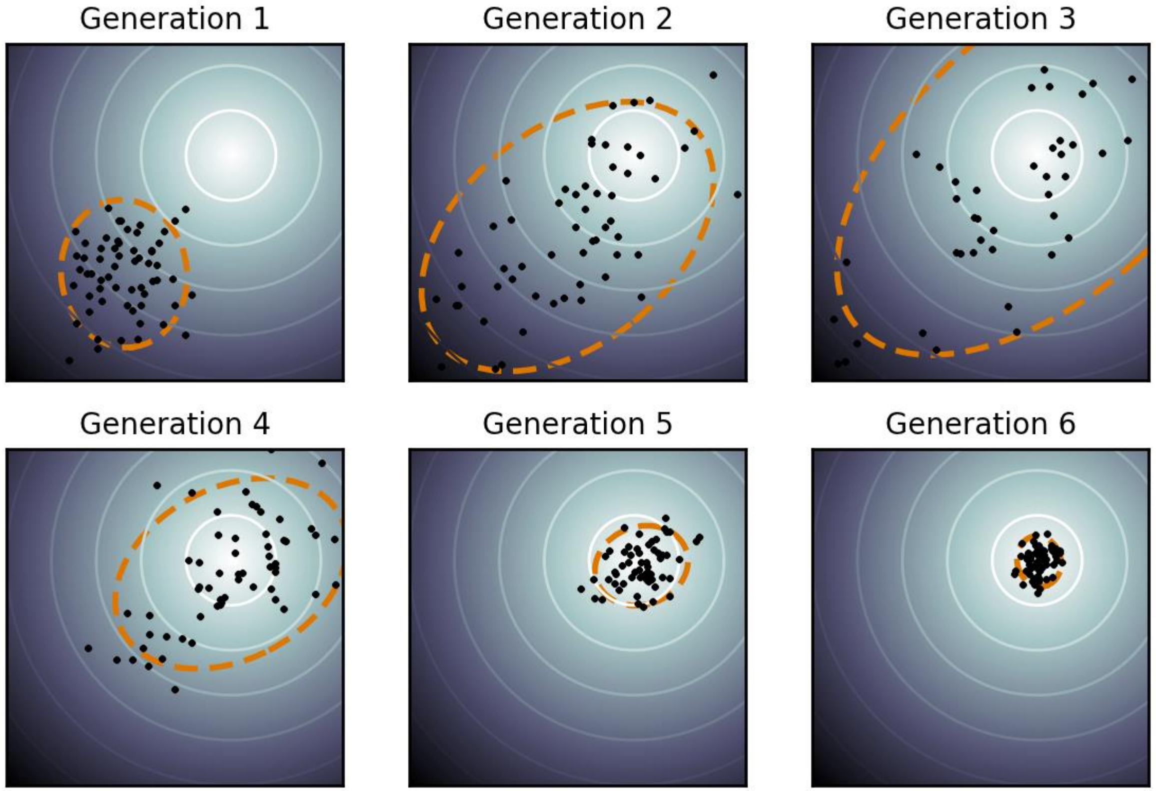
        - Update the estimation for $\boldsymbol{x}$
- Examples:
    - Bayesian optimization, Evolution strategies (e.g. CMA-ES), Stochastic optimization, Sequential Monte Carlo methods, ……

CMA-ES

- Covariance matrix adaptation evolution strategy (CMA-ES)
  - A widely adopted derivative-free method in character animation

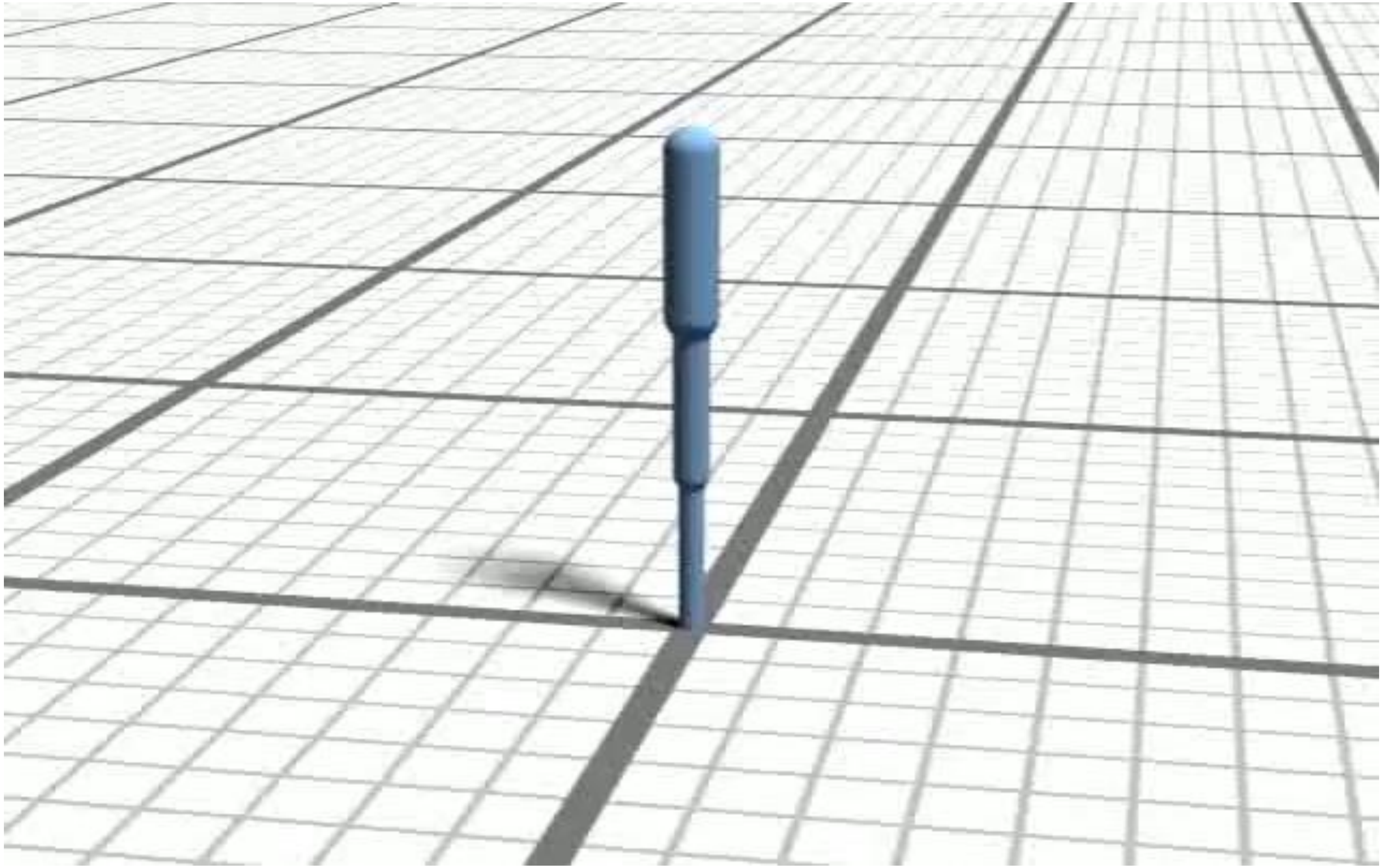


[Hansen 2006, The CMA evolution strategy: a comparing review]

Goal: find the variables $\boldsymbol{x}$ that optimize $f(\boldsymbol{x})$

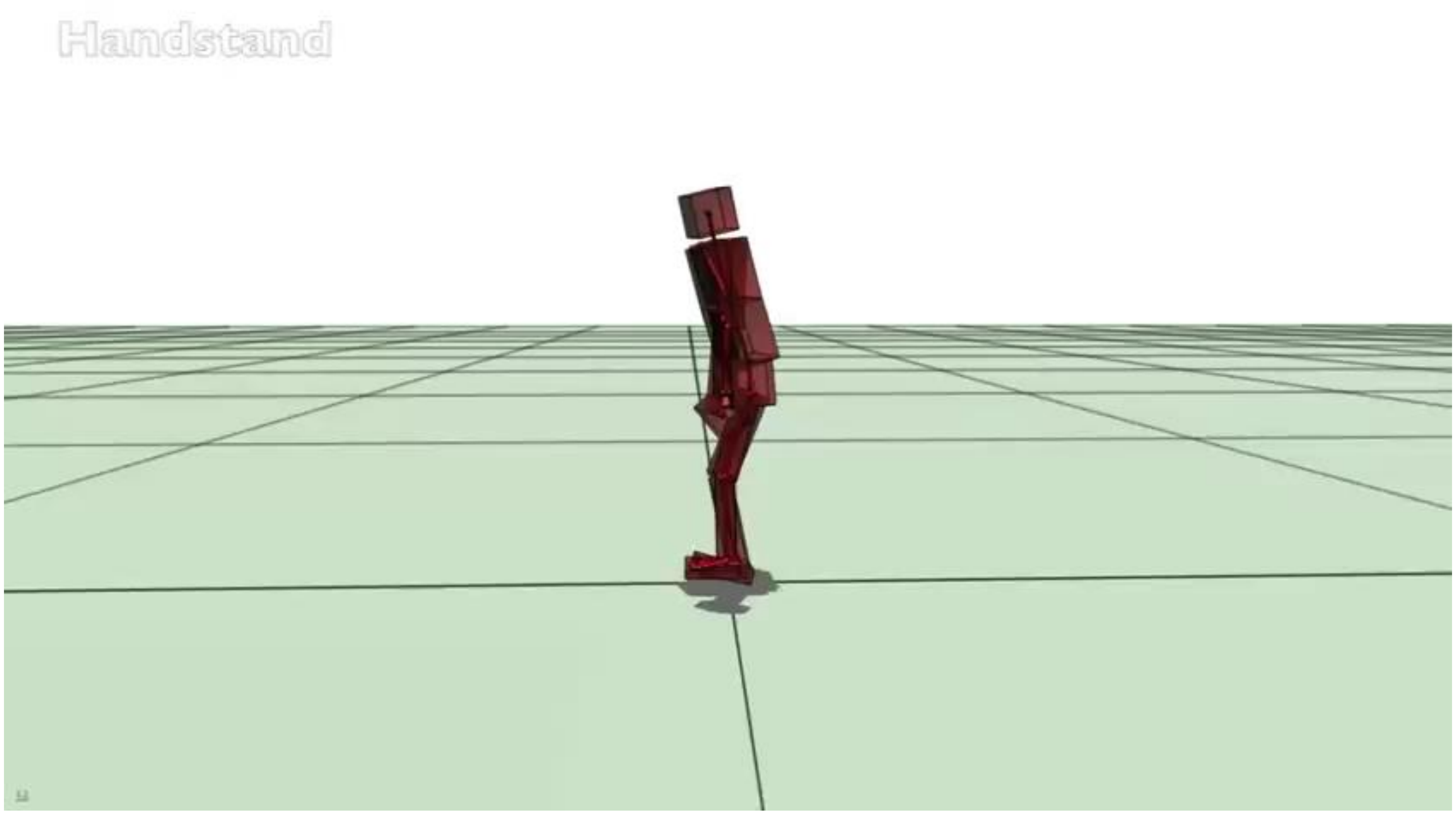
- Initialize Gaussian distribution $\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$
- Repeat:
  - sample candidate variables $\{\boldsymbol{x}_i\} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$
  - Evaluate the objective function $f_i = f(\boldsymbol{x}_i)$
    - Involve simulation and generate simulation trajectories
  - Sort $\{f_i\}$ and keep the top N elite samples
  - Update $\boldsymbol{\mu}, \Sigma$ according to the elite samples

CMA-ES



[Wampler and Popović 2009 - *Optimal Gait and Form for Animal Locomotion*]

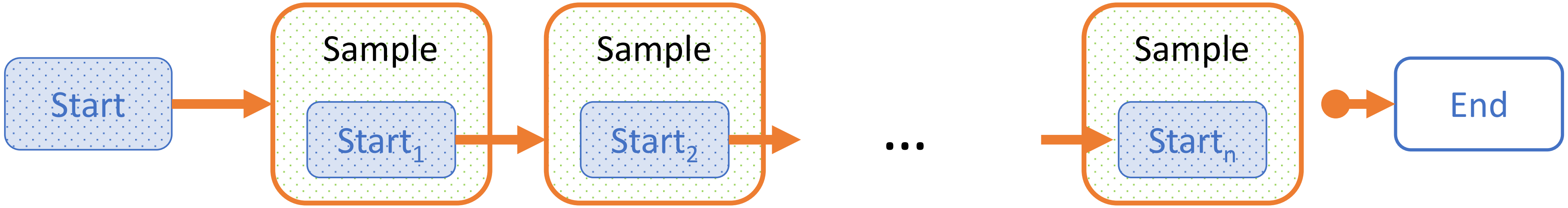
CMA-ES



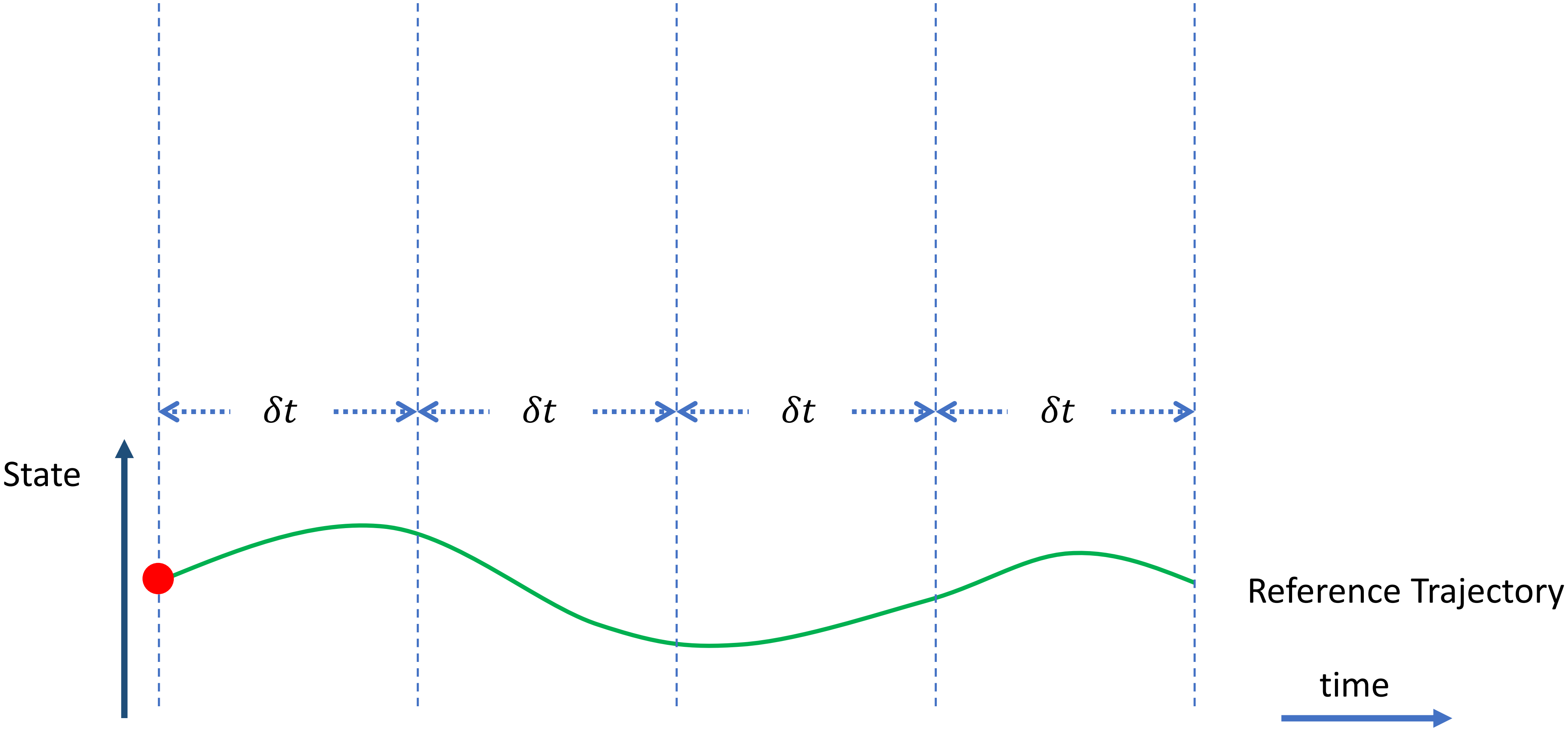
[Al Borno et al. 2013 - *Trajectory Optimization for Full-Body Movements with Complex Contacts*]

SAMCON

- **SA**mpling-based **M**otion **CON**trol [Liu et al. 2010, 2015]
  - Motion Clip → Open-loop control trajectory
  - A sequential Monte-Carlo method

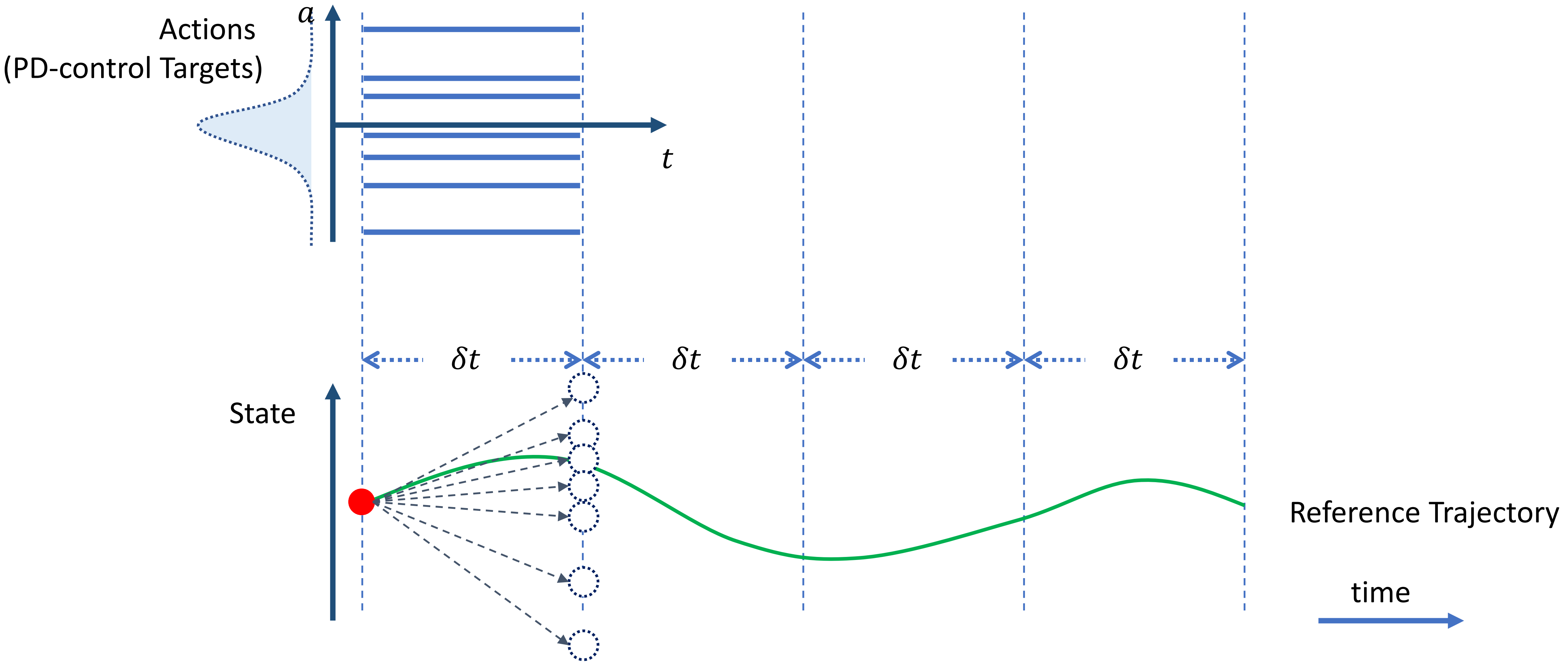


SAMCON

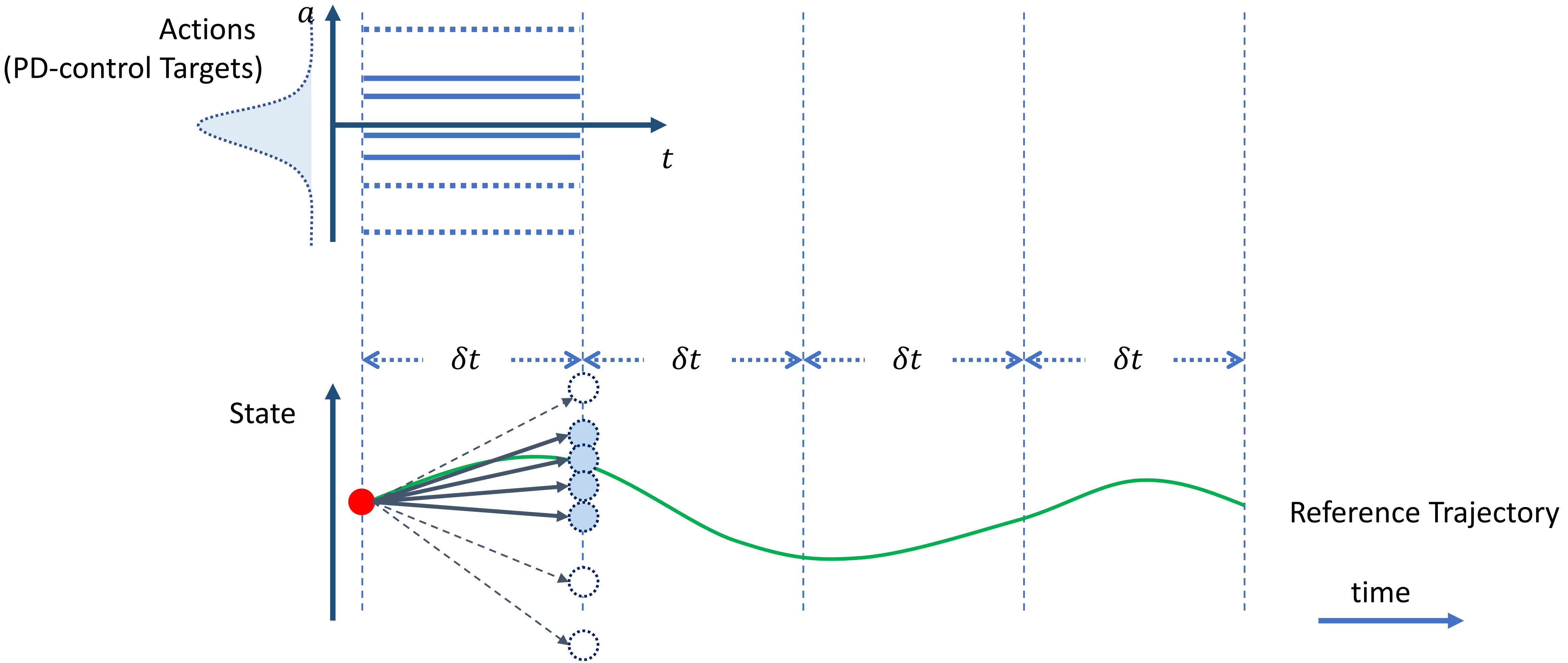


δt
δt
δt
δt
State
Reference Trajectory
time

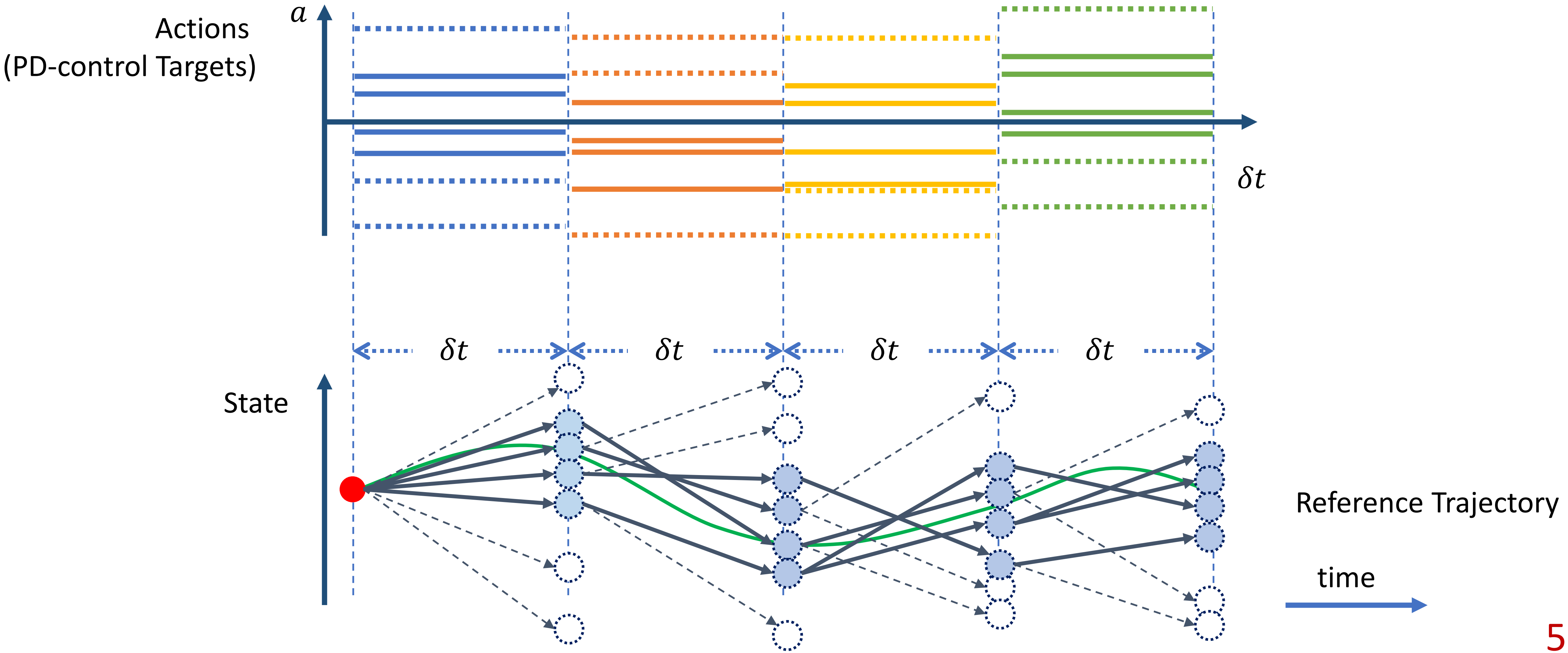
Sampling & Simulation



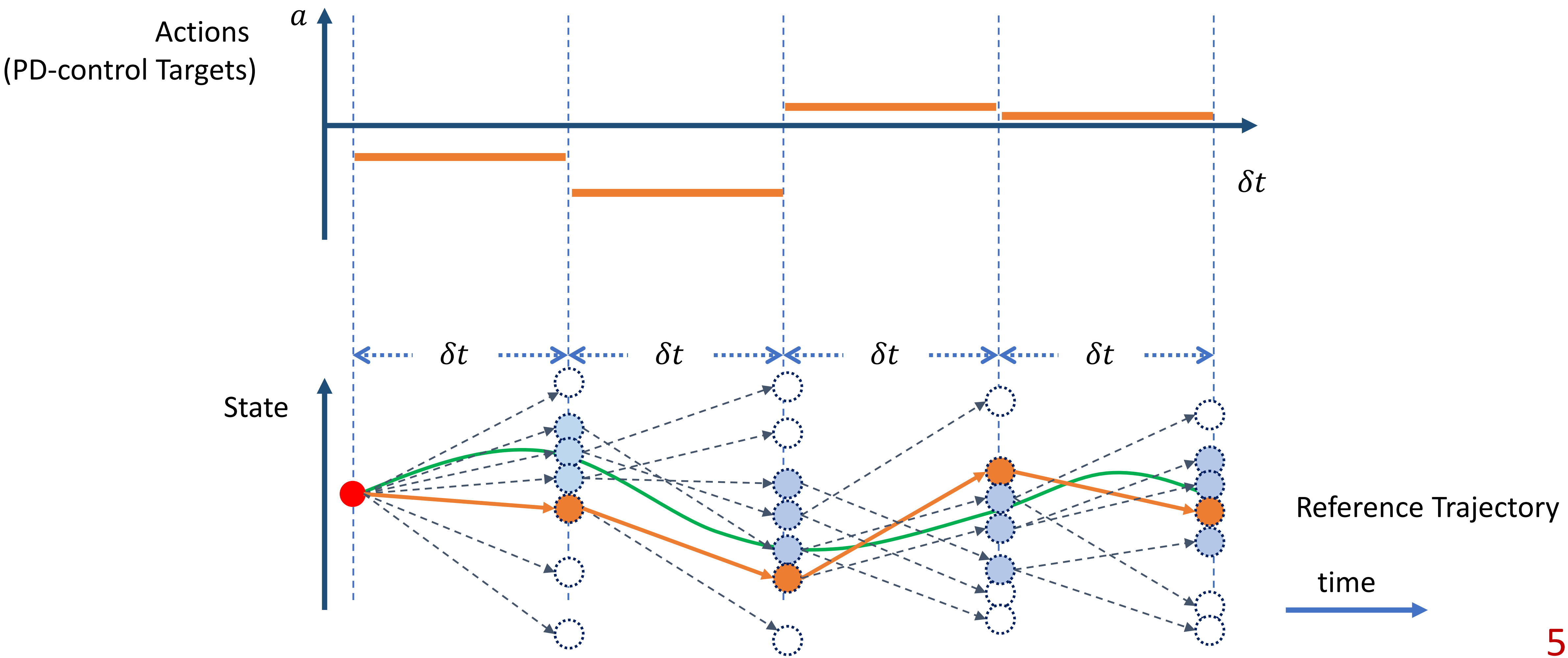
Sample Selection

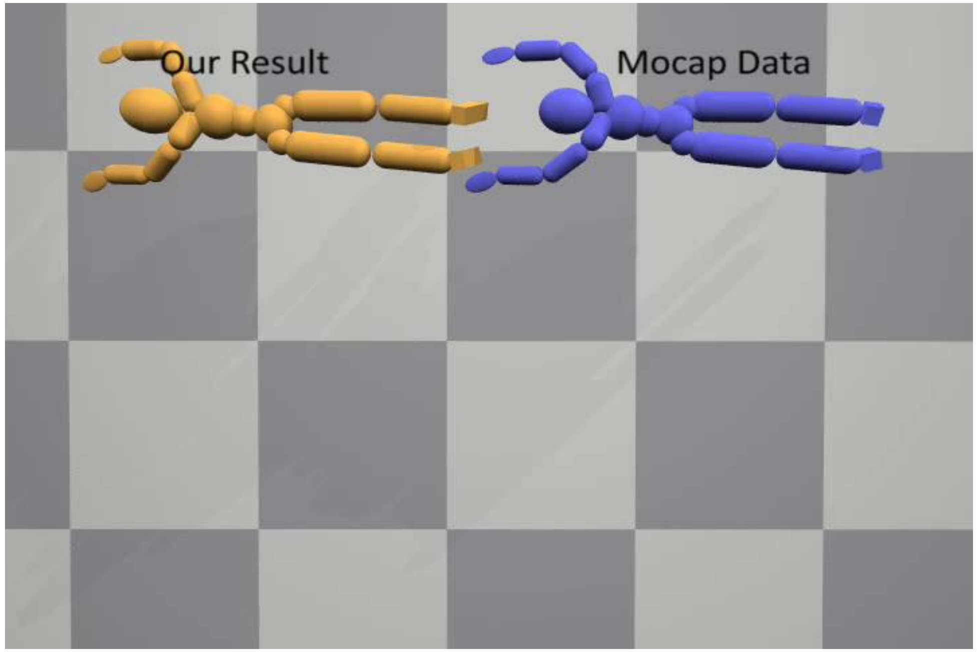


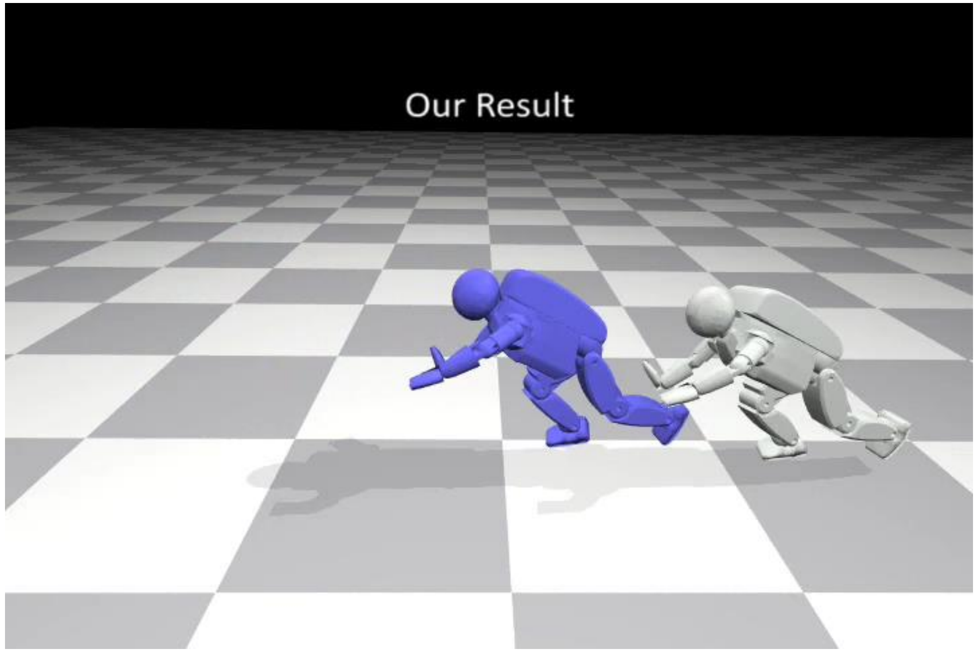
SAMCON Iterations

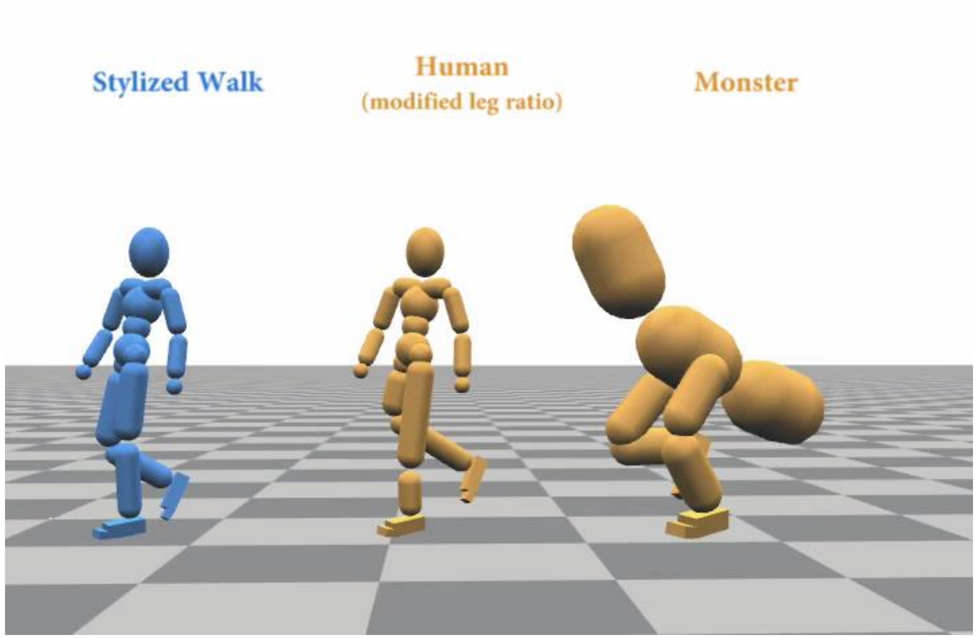


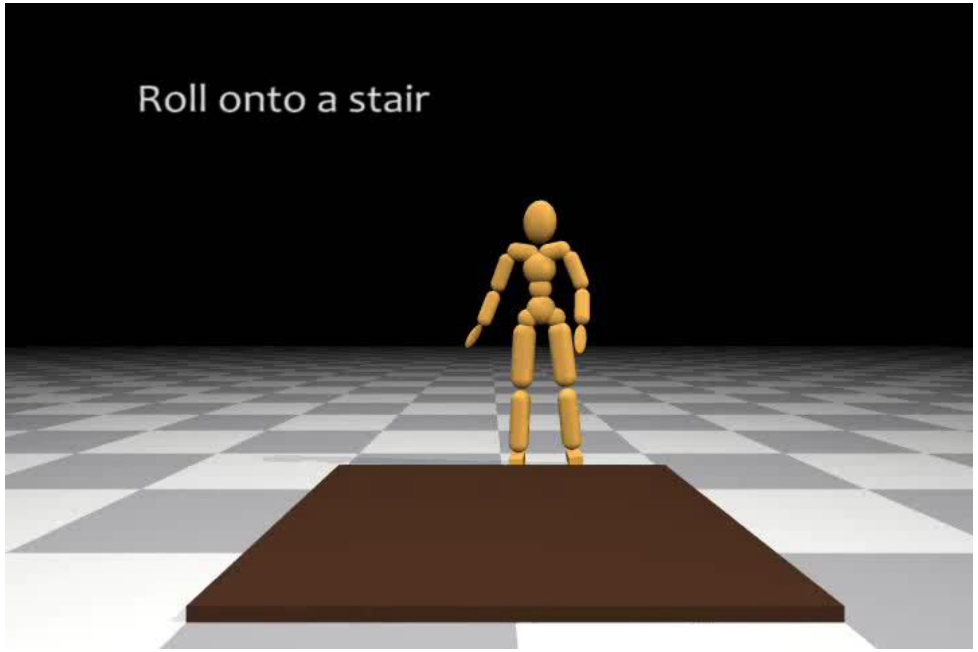
Constructed Open-loop Control Trajectory



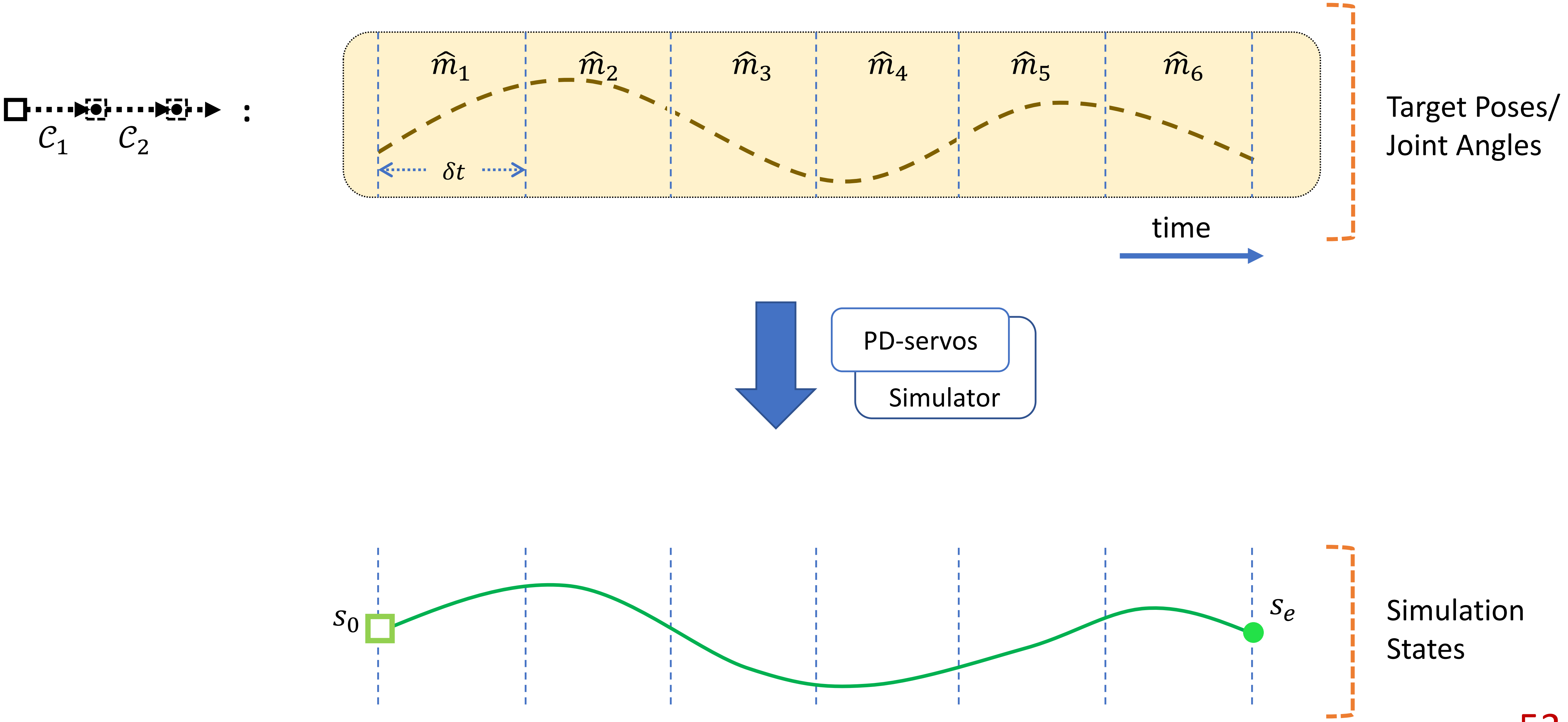

Our Result
Mocap Data


Our Result

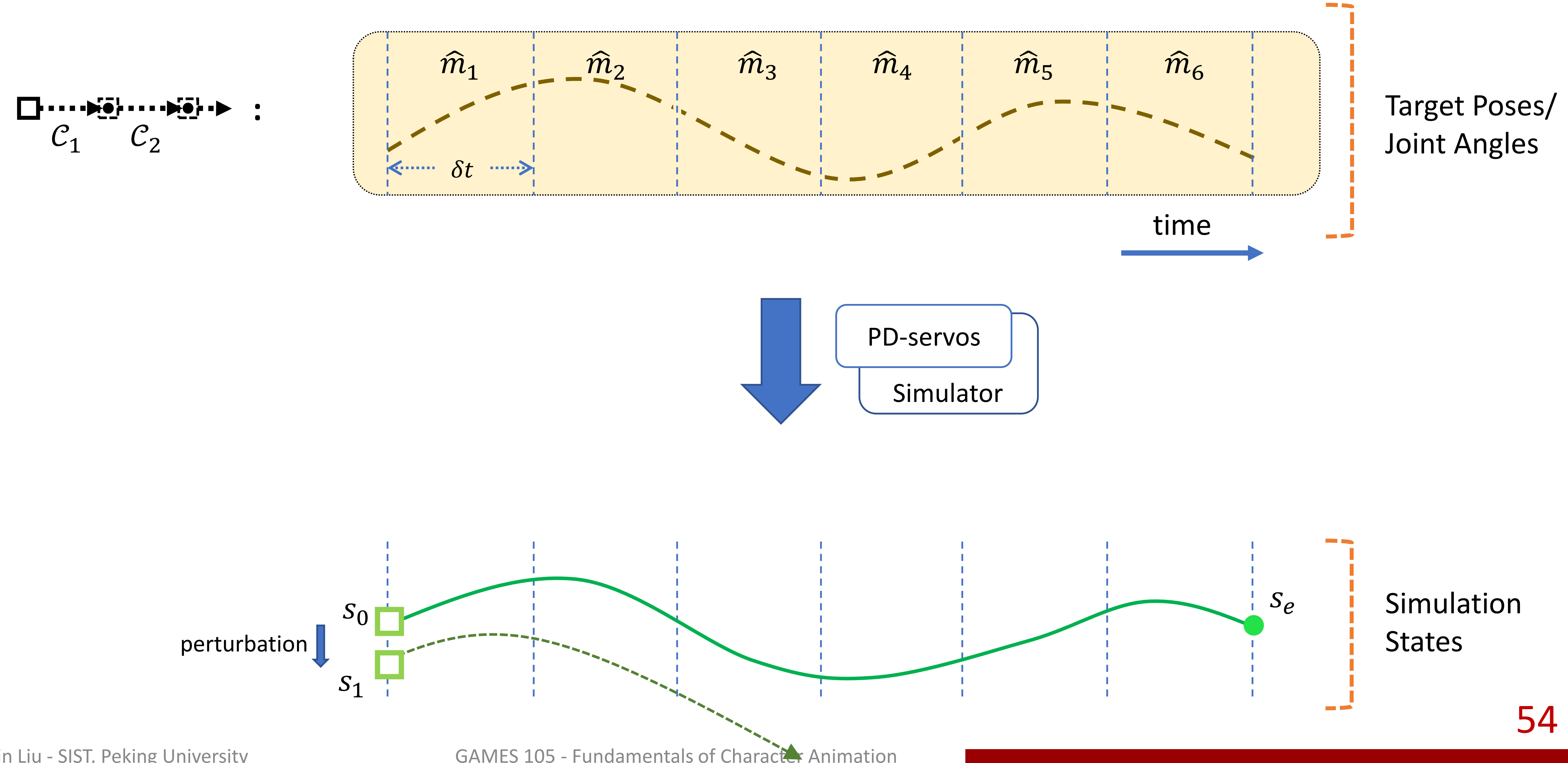

Stylized Walk
Human
(modified leg ratio)
Monster


Roll onto a stair

Feedforward Control



Feedforward Control



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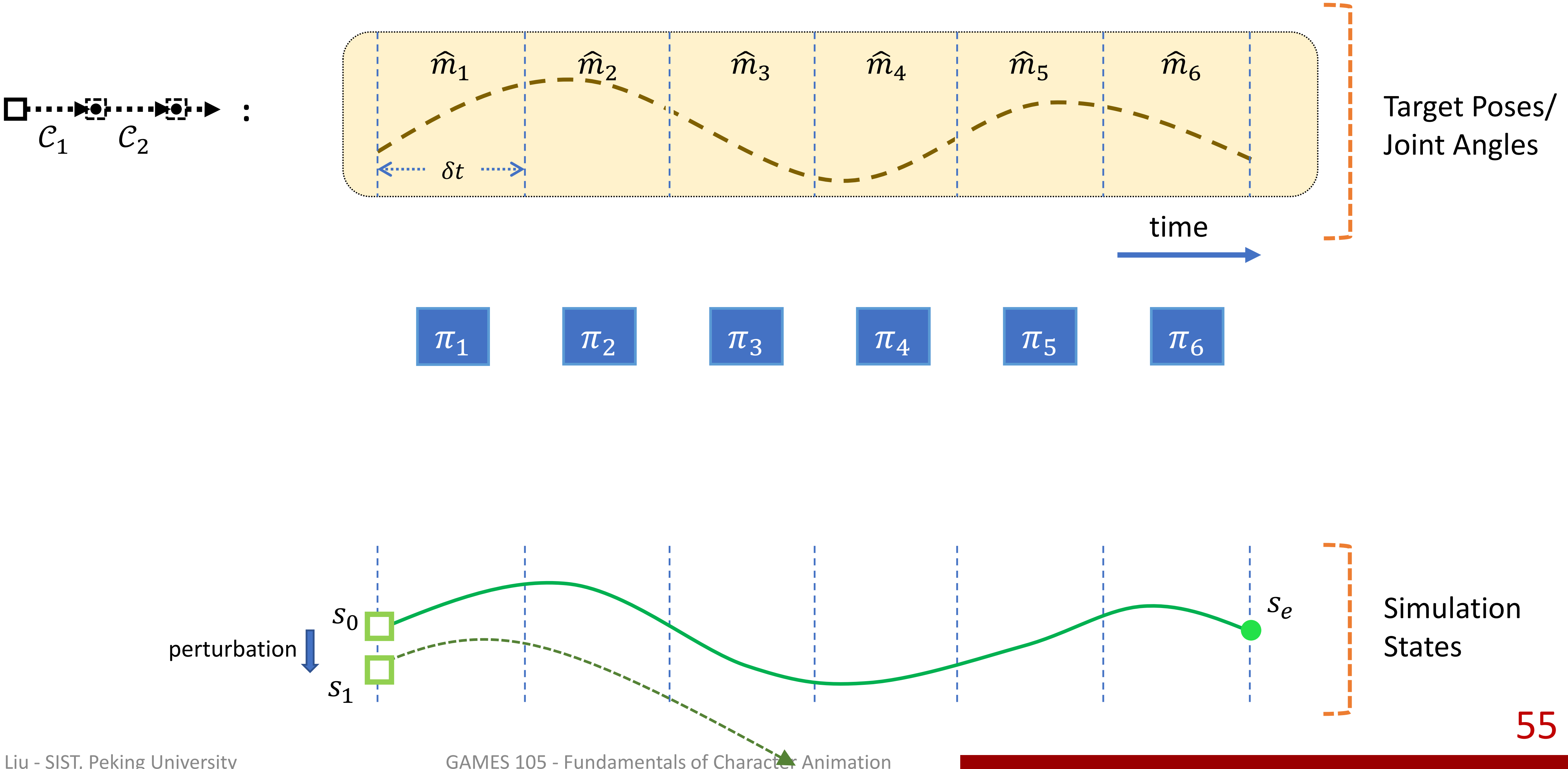
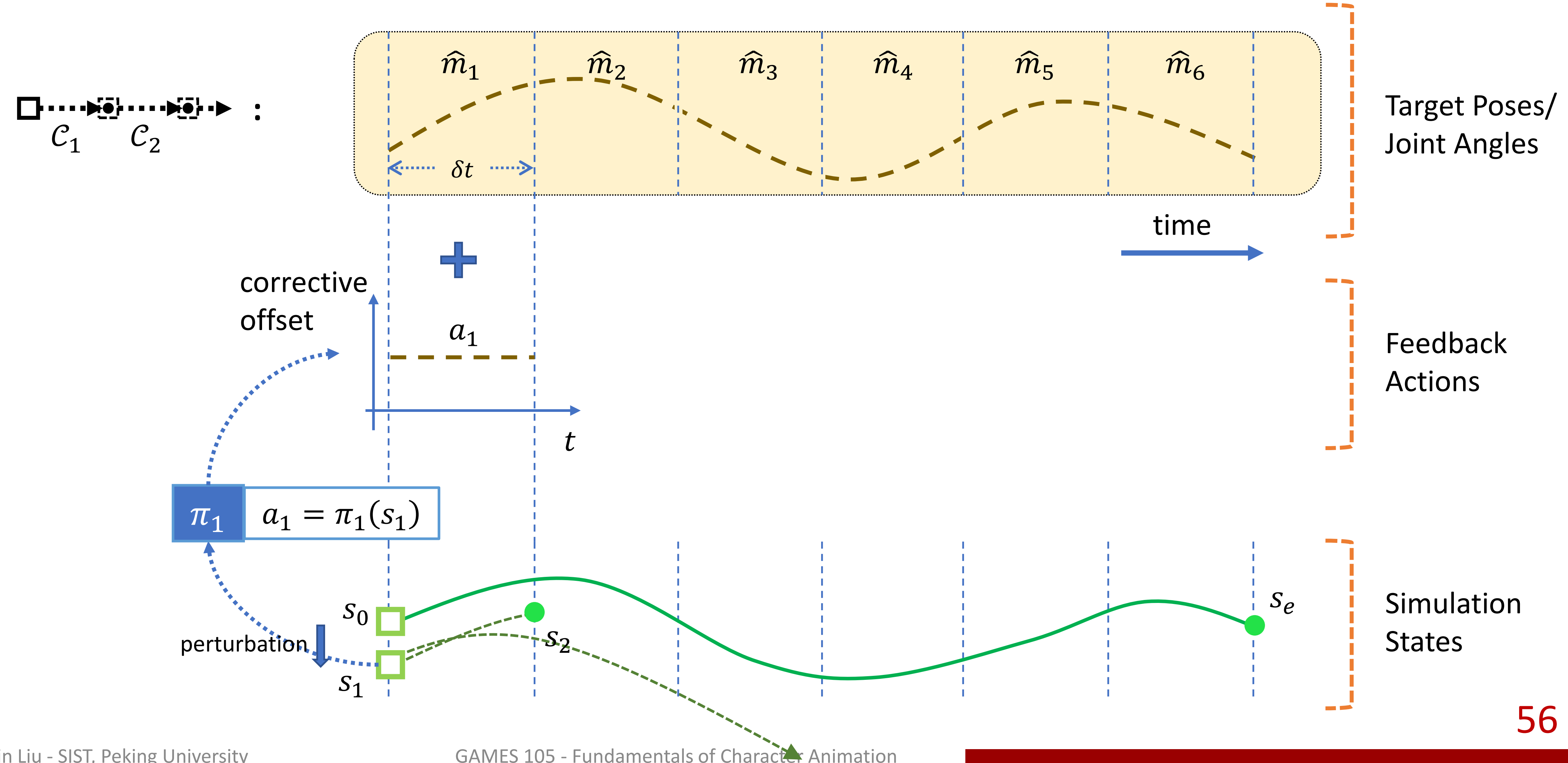
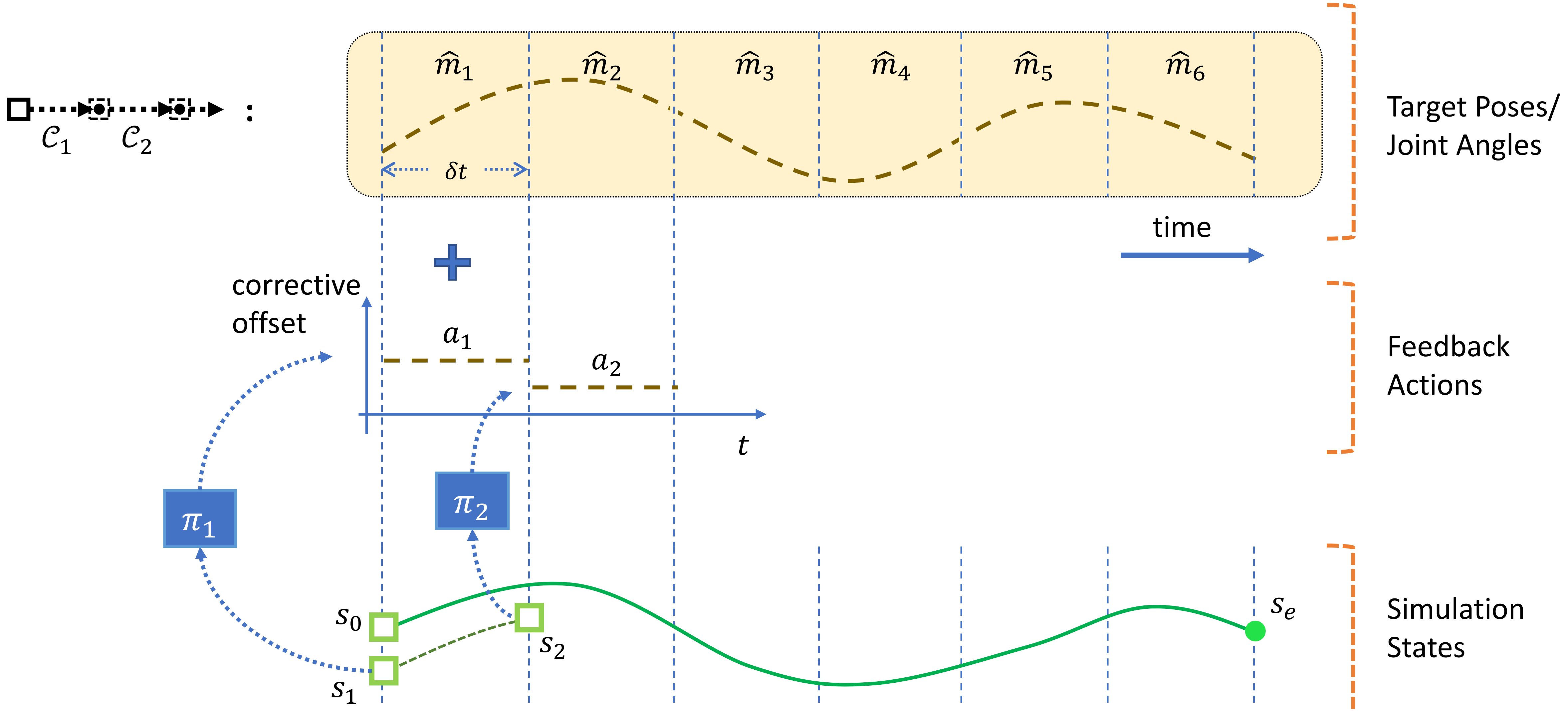


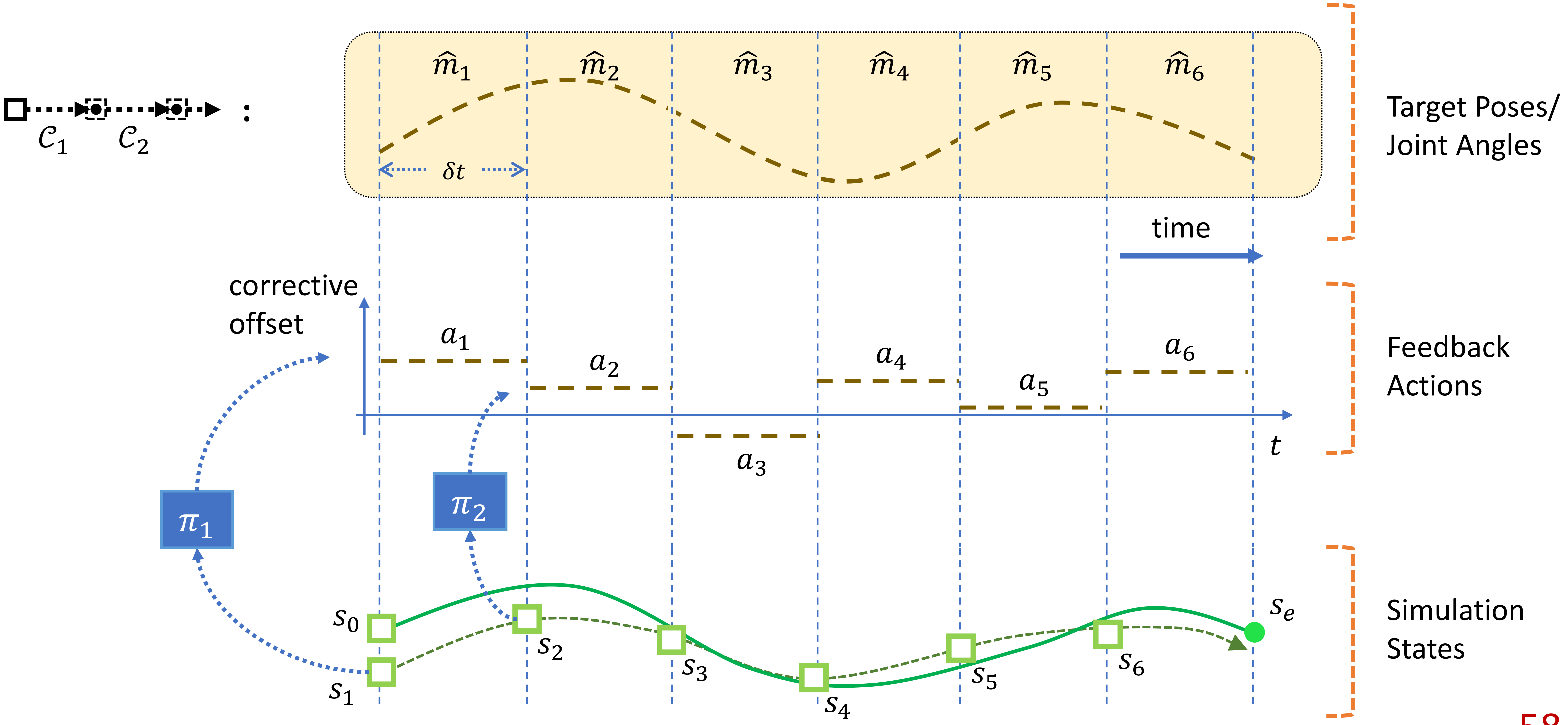
Feedback Control



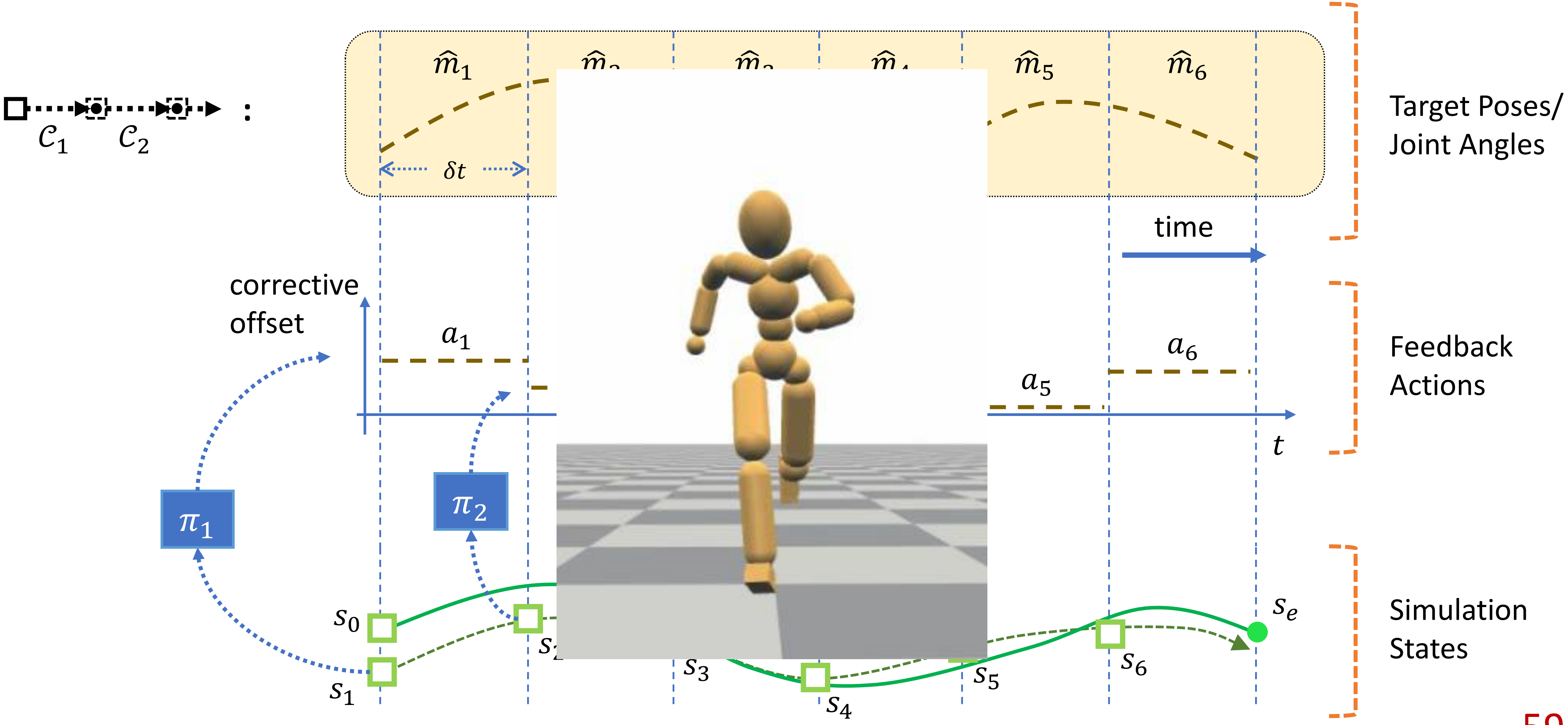
Feedback Control



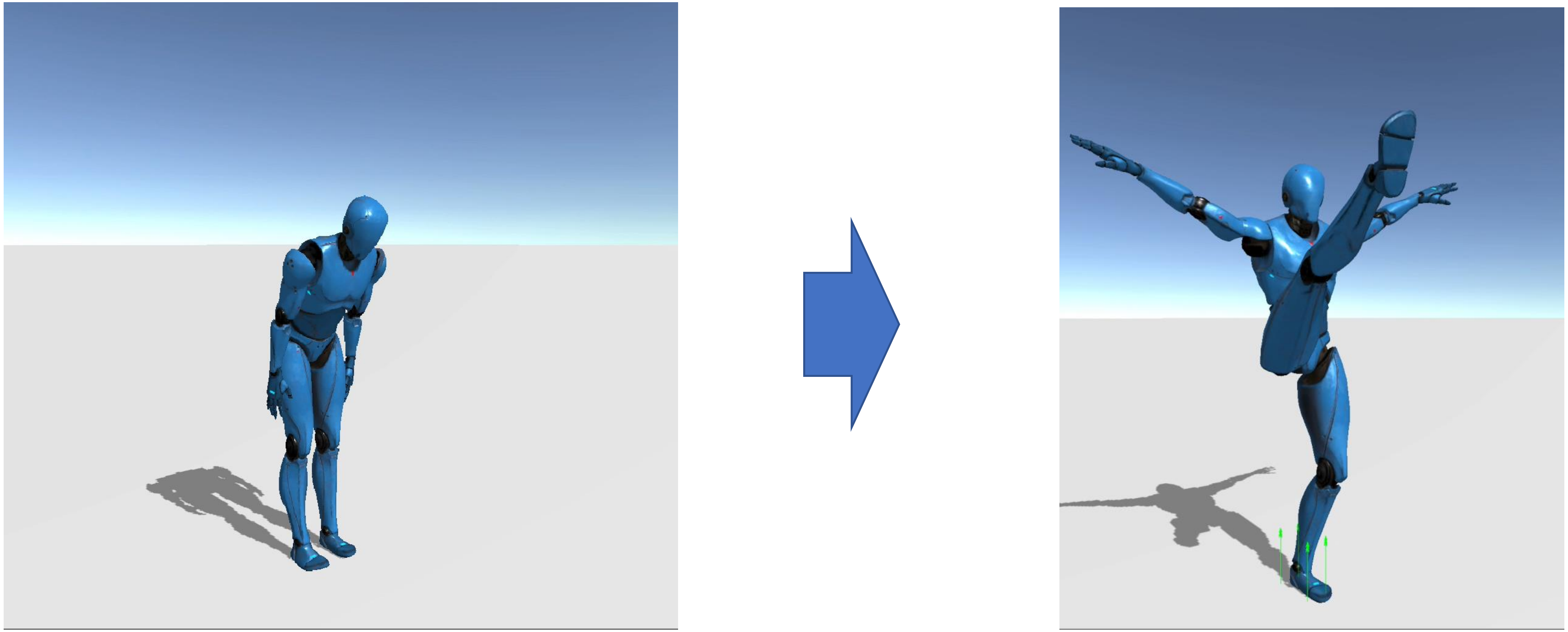
Feedback Control



Feedback Control

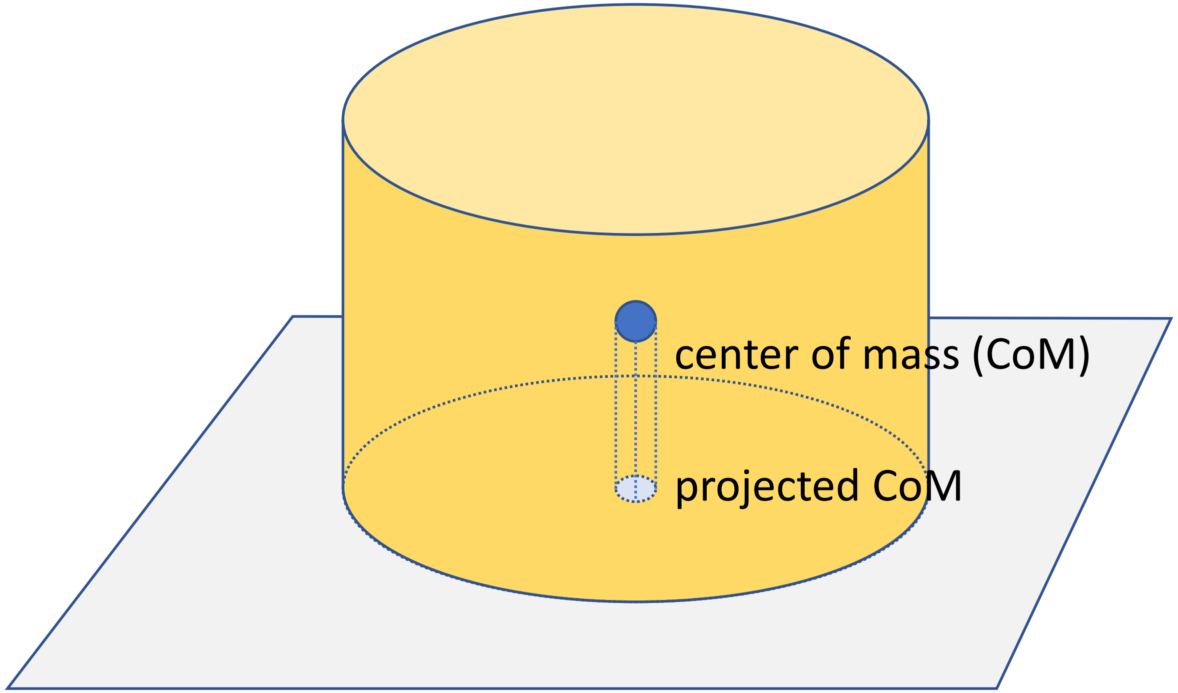


Static Balance



Static Balance

What is balance?



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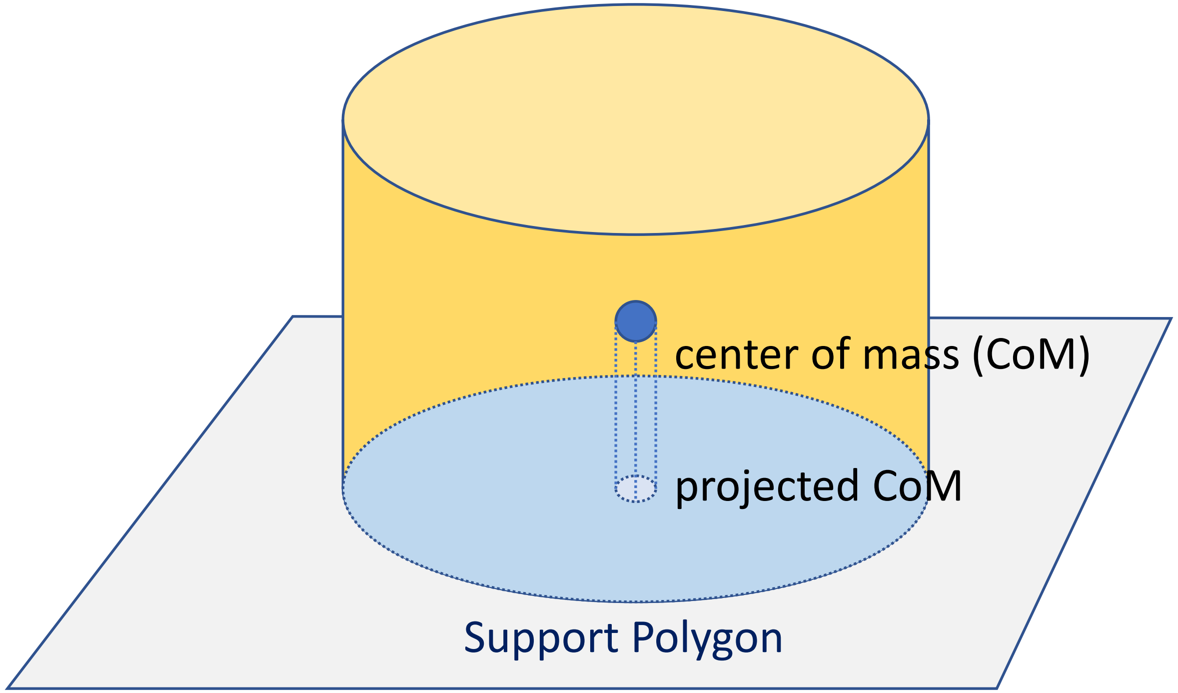

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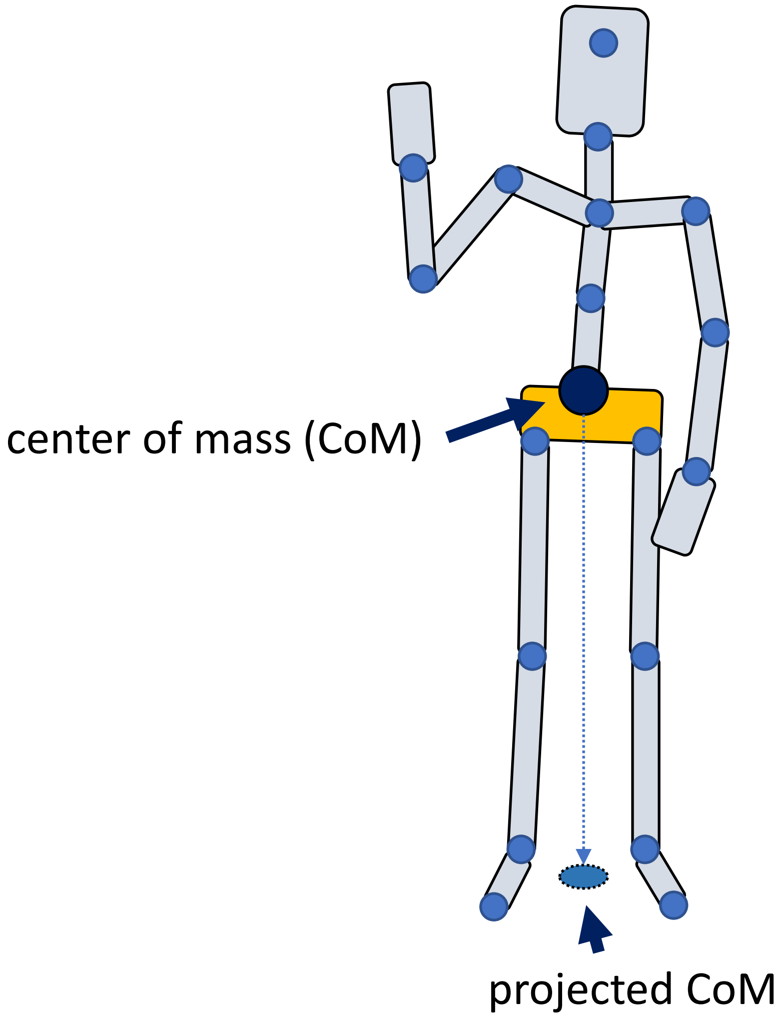

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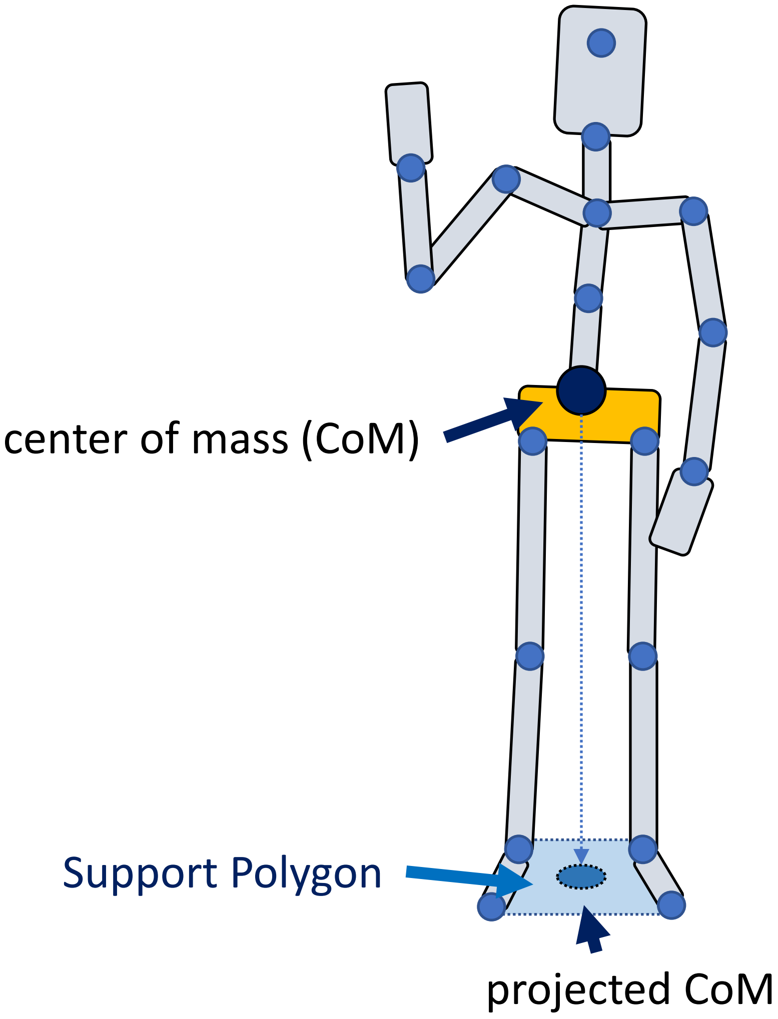
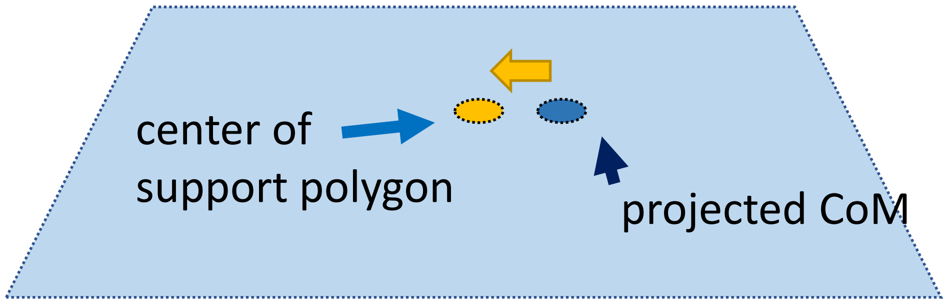
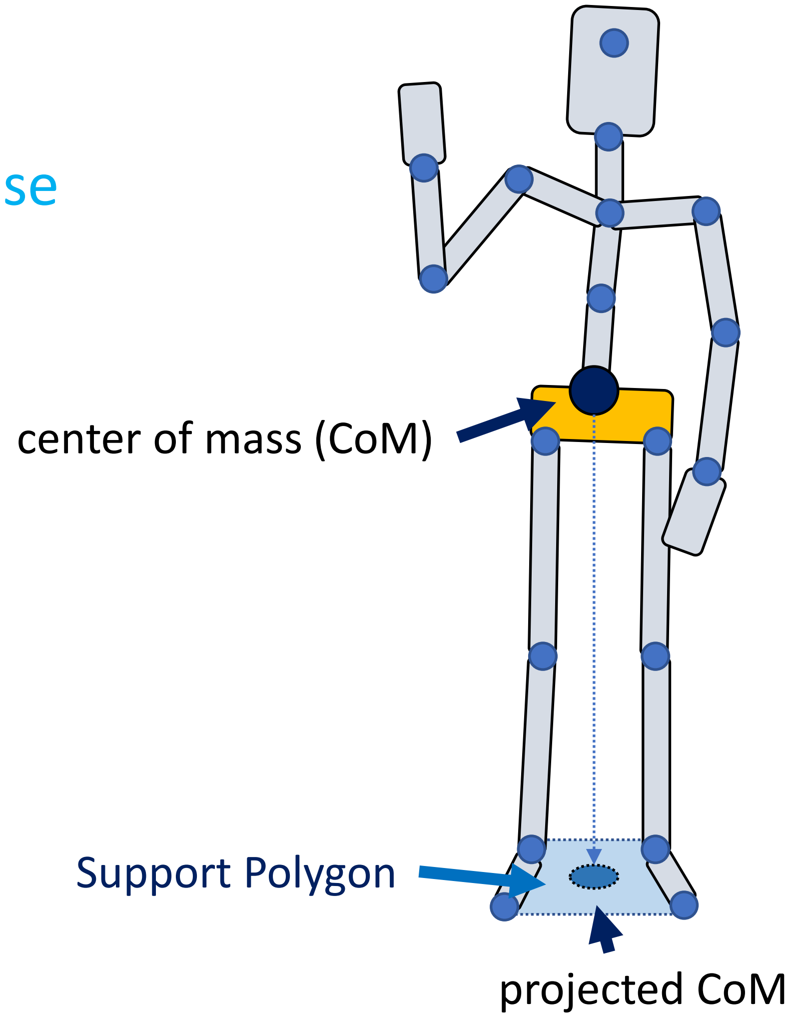


Static Balance

A simple strategy to maintain balance:

- Keep projected CoM close to the center of support polygon while tracking a standing pose





Static Balance

A simple strategy to maintain balance:

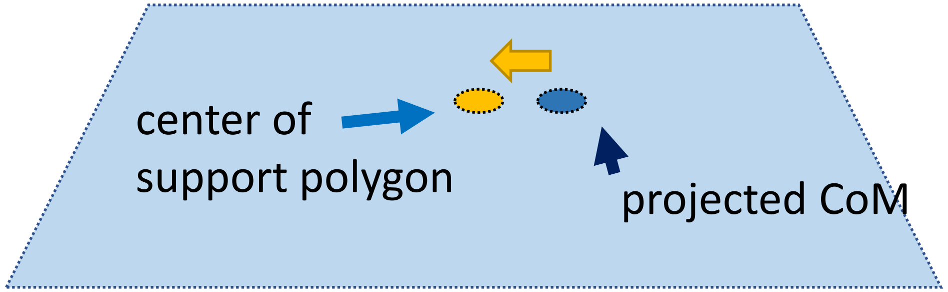
- Keep projected CoM close to the center of support polygon while tracking a standing pose
- Use PD control to compute feedback torque

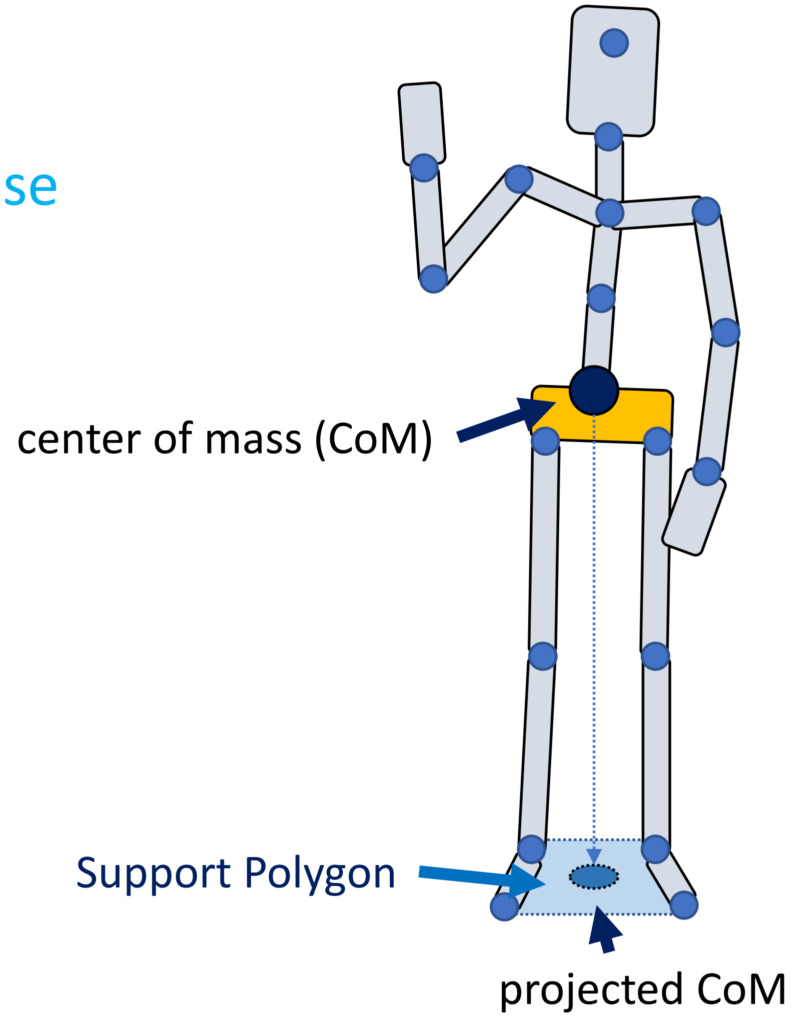
$$\tau = k_p(\bar{\boldsymbol{c}} - \boldsymbol{c}) - k_d\dot{\boldsymbol{c}}$$

CoM velocity

desired CoM position
(e.g. center of support polygon)

CoM position of
the character





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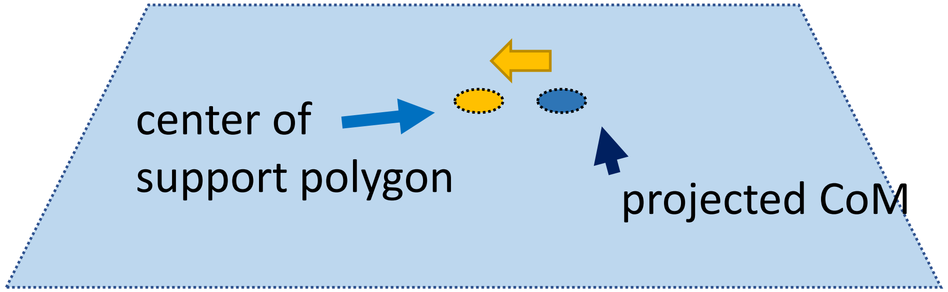
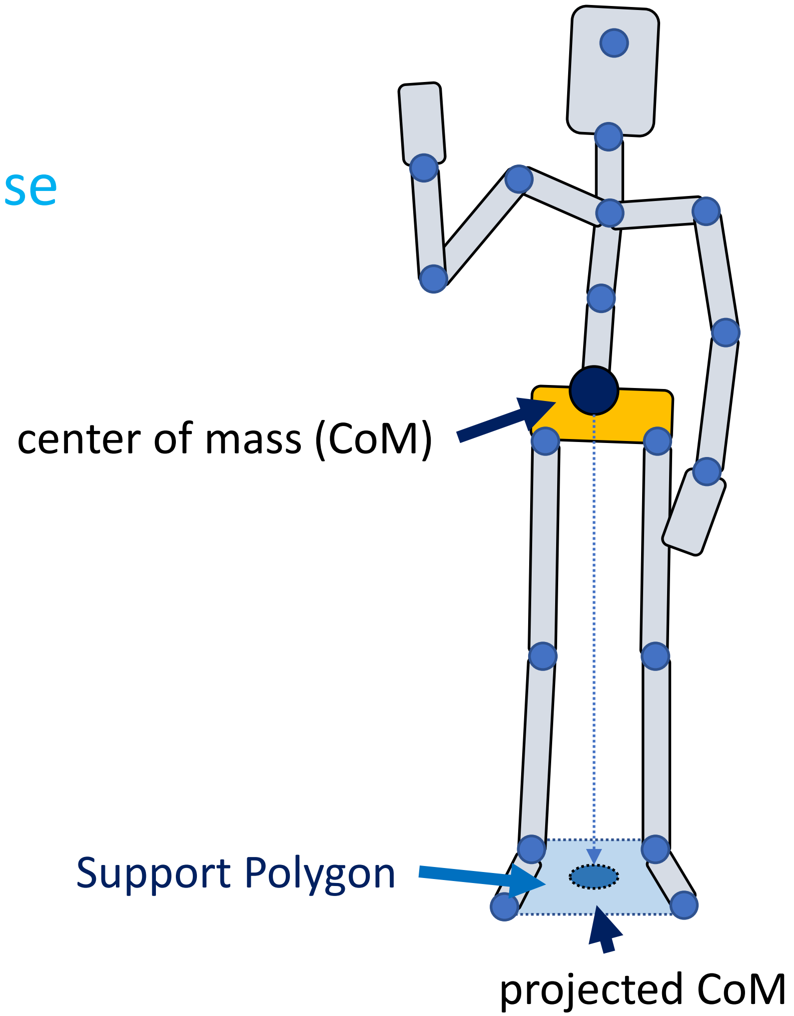
gain coefficient

damping coefficient

need to tune manually…





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- Keep projected CoM close to the center of support polygon while tracking a standing pose
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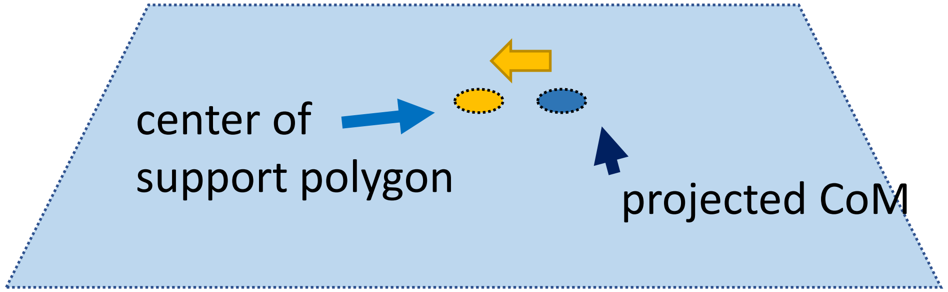
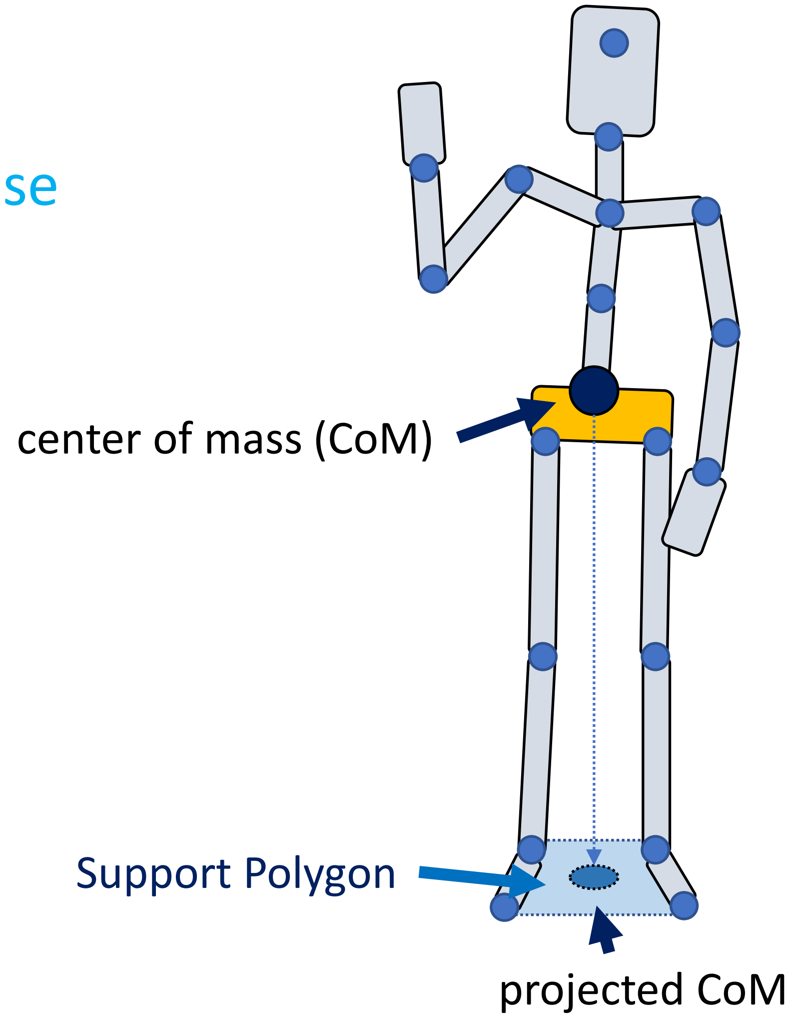
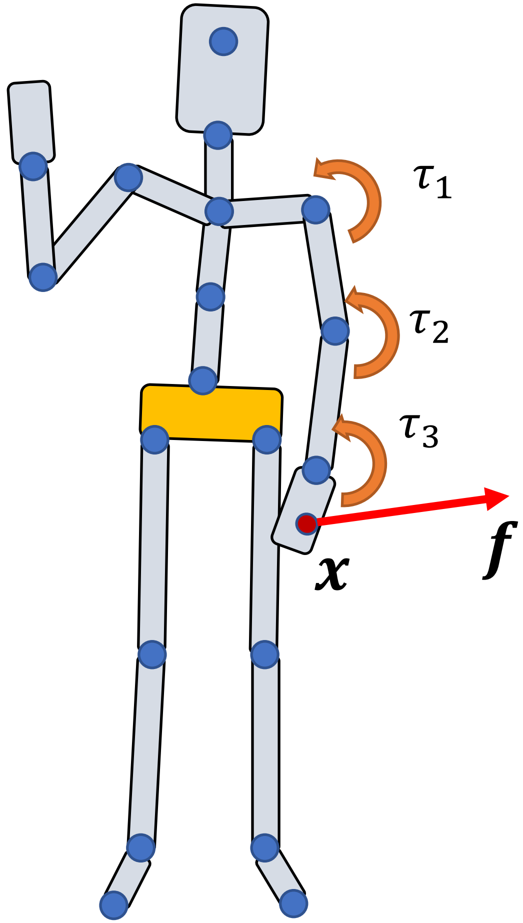
$$\tau = k_p(\bar{\boldsymbol{c}} - \boldsymbol{c}) - k_d\dot{\boldsymbol{c}}$$

- Apply the feedback torque at ankles (ankle strategy) or hips (hip strategy)





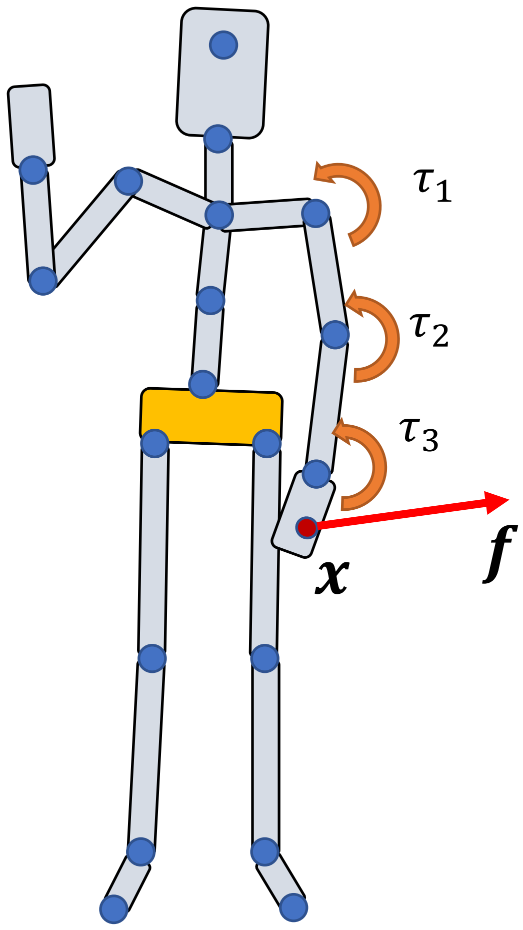
Jacobian Transpose Control



Can we use joint torques $\boldsymbol{\tau}_i$ to mimic the effect of a force $\boldsymbol{f}$ applied at $\boldsymbol{x}$

- Note that the desired force $\boldsymbol{f}$ is not actually applied
- Also called "virtual force"

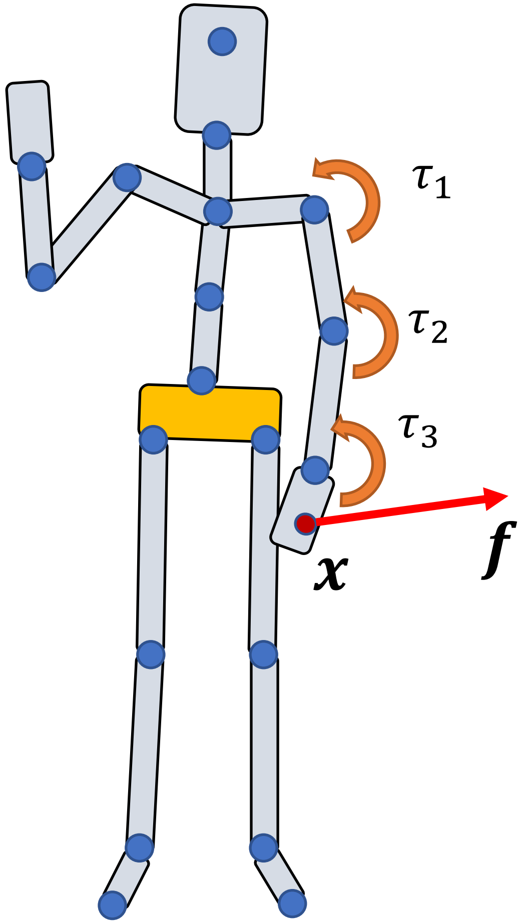
Jacobian Transpose Control



Can we use joint torques $\boldsymbol{\tau}_i$ to mimic the **effect** of a force $\boldsymbol{f}$ applied at $\boldsymbol{x}$

Make $\boldsymbol{f}$ and $\boldsymbol{\tau}_i$ done the same work

Jacobian Transpose Control

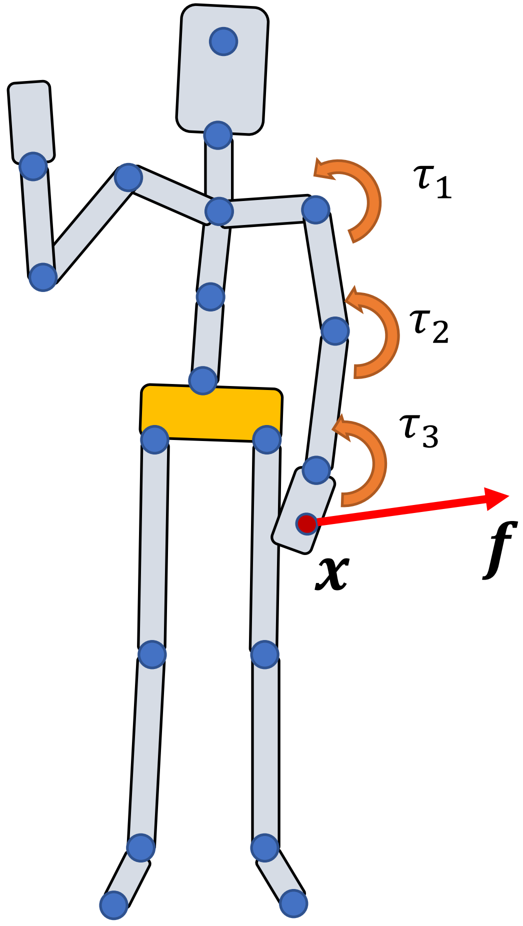


Can we use joint torques $\boldsymbol{\tau}_i$ to mimic the **effect** of a force $\boldsymbol{f}$ applied at $\boldsymbol{x}$

Make $\boldsymbol{f}$ and $\boldsymbol{\tau}_i$ done the same power

$$P = \boldsymbol{f}^T \dot{\boldsymbol{x}} = \boldsymbol{\tau}^T \dot{\boldsymbol{\theta}}$$

Jacobian Transpose Control



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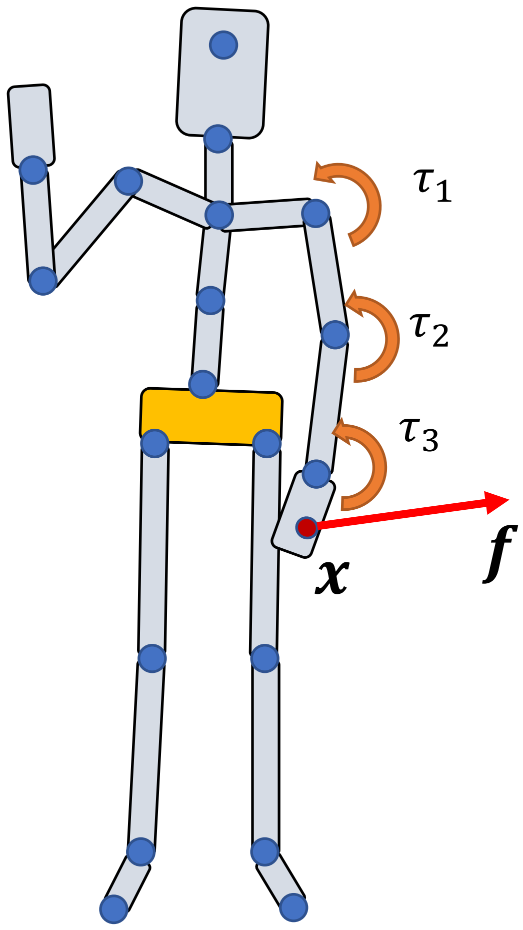
$$P = \boldsymbol{f}^T \dot{\boldsymbol{x}} = \boldsymbol{\tau}^T \dot{\boldsymbol{\theta}}$$

Forward kinematics $\boldsymbol{x} = g(\boldsymbol{\theta}) \Rightarrow \dot{\boldsymbol{x}} = J\dot{\boldsymbol{\theta}}$

$$J = \frac{\partial g}{\partial \boldsymbol{\theta}}$$

Jacobian Transpose Control



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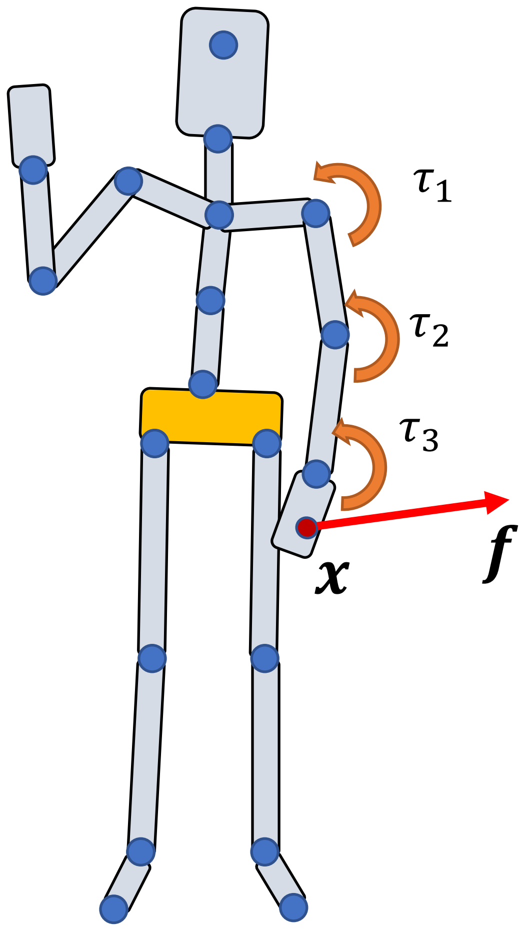
Jacobian Transpose Control



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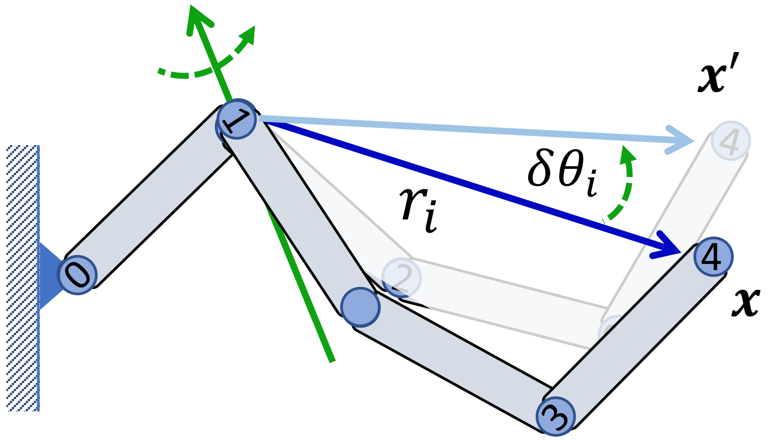
$$\boldsymbol{\tau} = J^T \boldsymbol{f}$$

$$J = \frac{\partial g}{\partial \boldsymbol{\theta}}$$

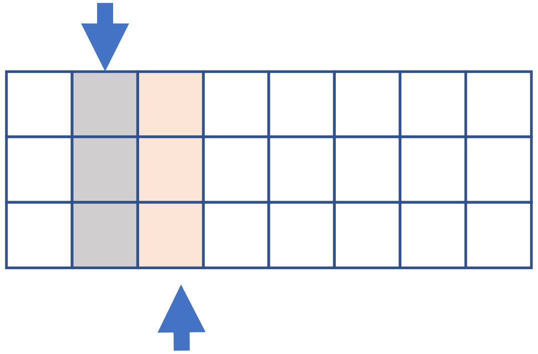
Recall: Geometric Approach

$$J = \frac{\partial g}{\partial \boldsymbol{\theta}} = \begin{pmatrix} \frac{\partial g}{\partial \theta_0} & \frac{\partial g}{\partial \theta_1} & \cdots & \frac{\partial g}{\partial \theta_n} \end{pmatrix}$$

Assuming all joints are hinge joint

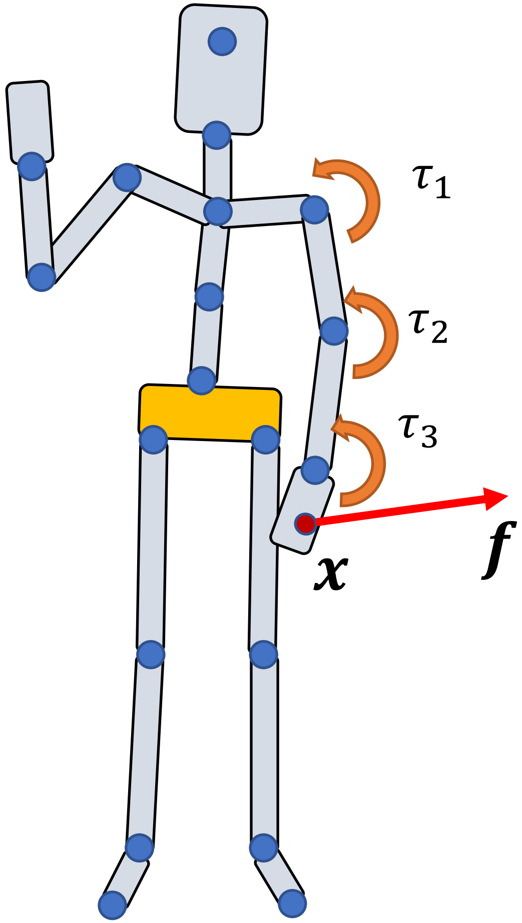


$$\frac{\partial g}{\partial \theta_1} = \boldsymbol{a}_1 \times \boldsymbol{r}_1$$



$$\frac{\partial f}{\partial \theta_2} = \boldsymbol{a}_2 \times \boldsymbol{r}_2$$

Jacobian Transpose Control



Can we use joint torques $\boldsymbol{\tau}_i$ to mimic the **effect** of a force $\boldsymbol{f}$ applied at $\boldsymbol{x}$

Make $\boldsymbol{f}$ and $\boldsymbol{\tau}_i$ done the same power

$$\boldsymbol{\tau} = J^T \boldsymbol{f} \qquad J = \frac{\partial g}{\partial \boldsymbol{\theta}}$$

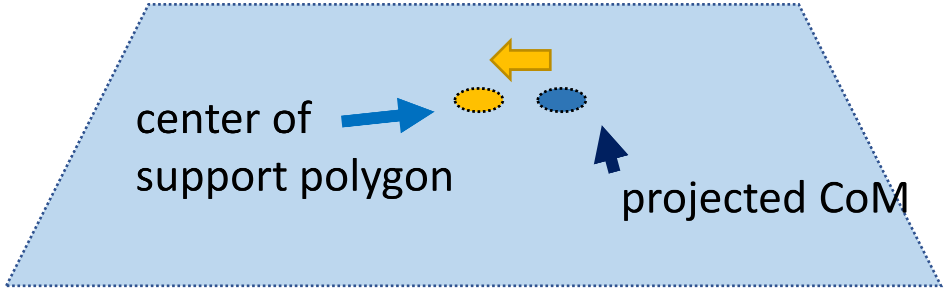
$$\boldsymbol{\tau}_i = (\boldsymbol{x} - \boldsymbol{p}_i) \times \boldsymbol{f}$$

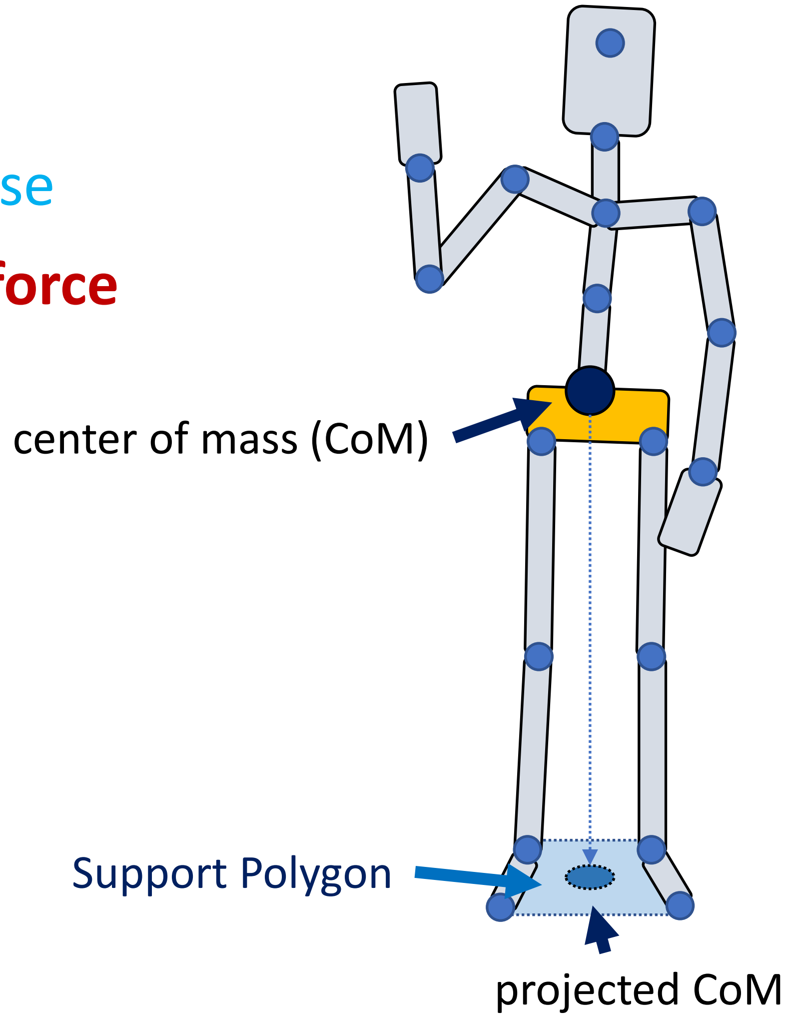
Static Balance

A simple strategy to maintain balance:

- Keep projected CoM close to the center of support polygon while tracking a standing pose
- Use PD control to compute feedback **virtual force**

$$f = k_p(\bar{\boldsymbol{c}} - \boldsymbol{c}) - k_d\dot{\boldsymbol{c}}$$





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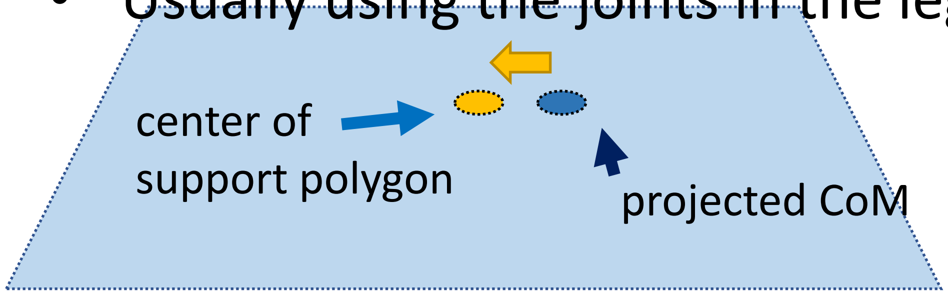
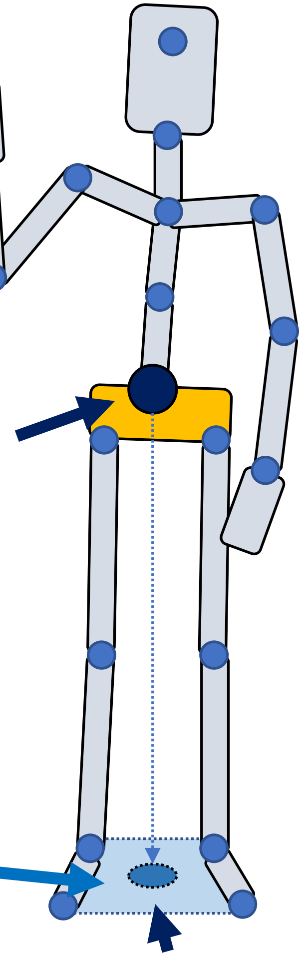
- Assuming f is applied to the CoM, compute necessary joint torques using Jacobian transpose control to achieve it
  - Usually using the joints in the legs

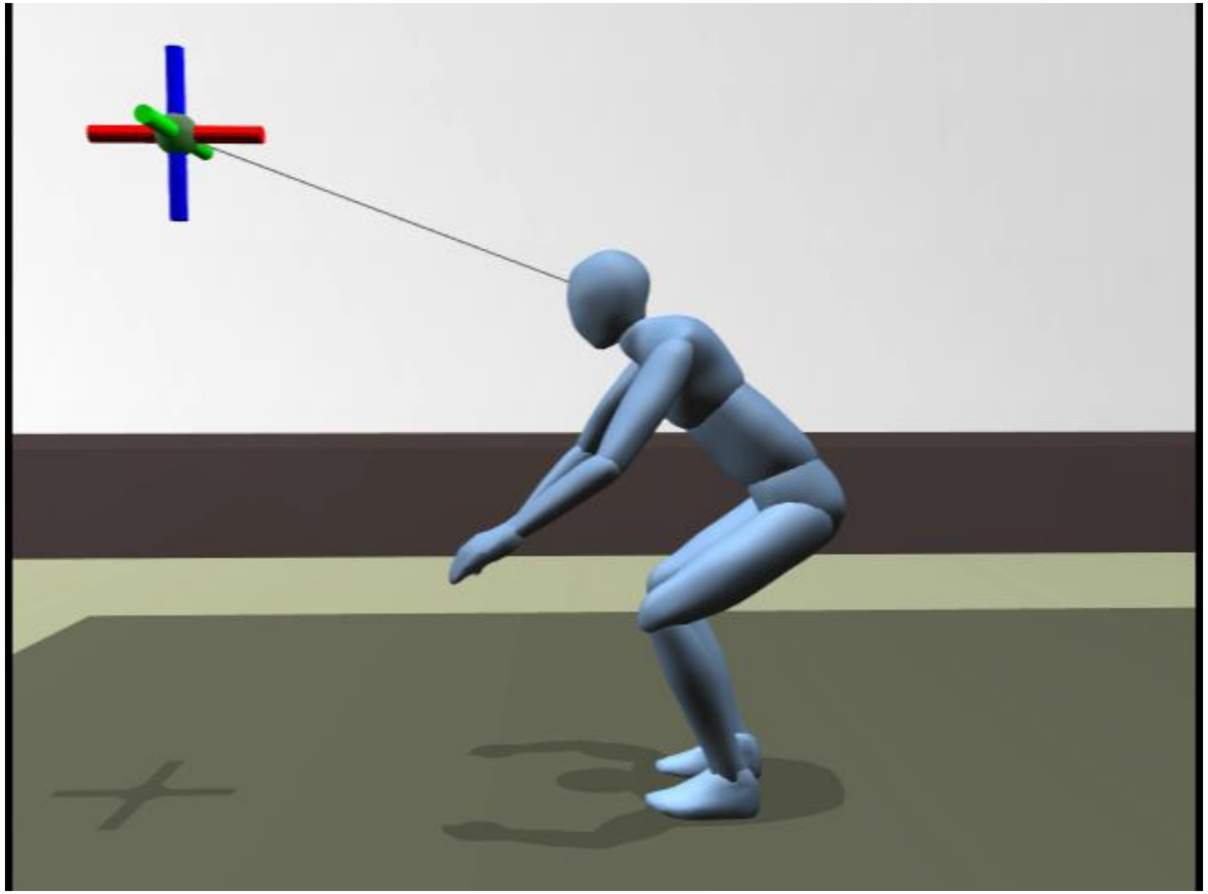




Static Balance

A fancier strategy:

- Mocap tracking as an objective function
- Controlling both the CoM position/momentum and the angular momentum
- Solve a one-step optimization problem to compute joint torques



[Macchietto et al. 2009 - Momentum Control for Balance]

Outline

- More about PD (Proportional-Derivative) control
  - Stable PD control
- Feedforward Motion Control
  - Trajectory optimization
- Feedback Motion Control
  - Static balance
  - Dynamic balance?

Questions?