GAMES 105
Fundamentals of Character Animation

Lecture 09

Actuating Simulated Characters

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GAMES105 课程交流

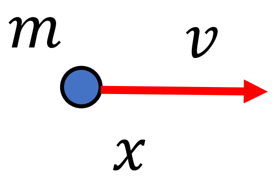
VCL @ PKU

Outline

- Simulating & Actuating Characters
  - Joint torques

- PD (Proportional-Derivative) control

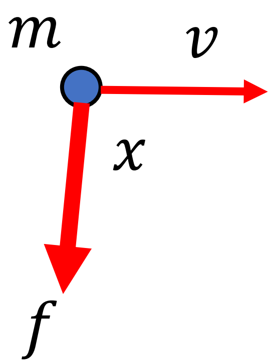
Recap: Dynamics of a Point Mass



x, v

$p = mv$

Recap: Dynamics of a Point Mass

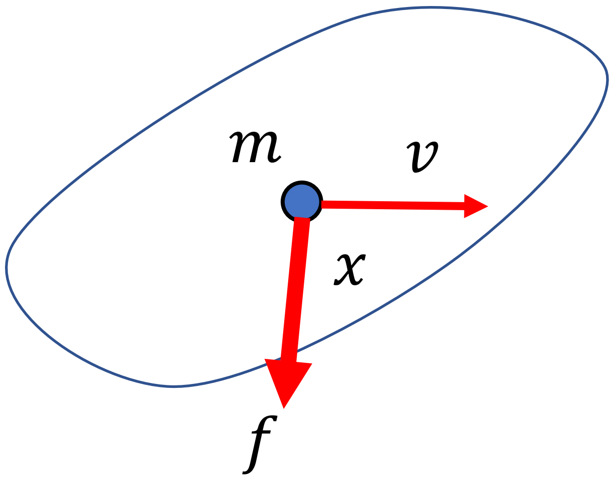


$$x, v$$

$$p = mv$$

Newton's Second Law: $\frac{dp}{dt} = f \quad \Rightarrow \quad m\dot{v} = f$

Recap: Rigid Body Dynamics



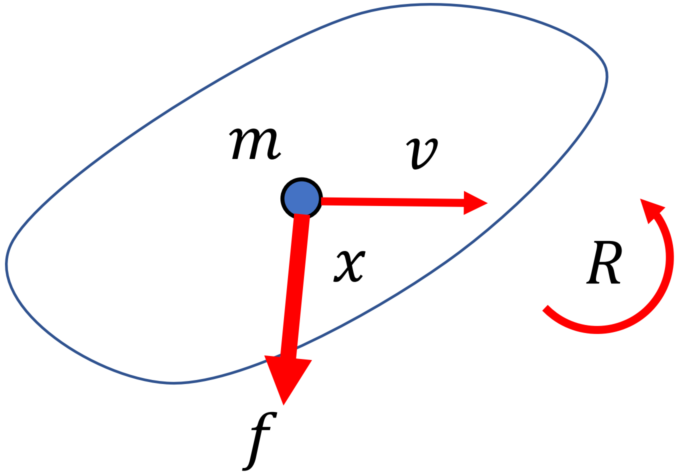
Linear

x, v

$p = mv$

Newton's Second Law: $\frac{dp}{dt} = f$ ➡ $m\dot{v} = f$

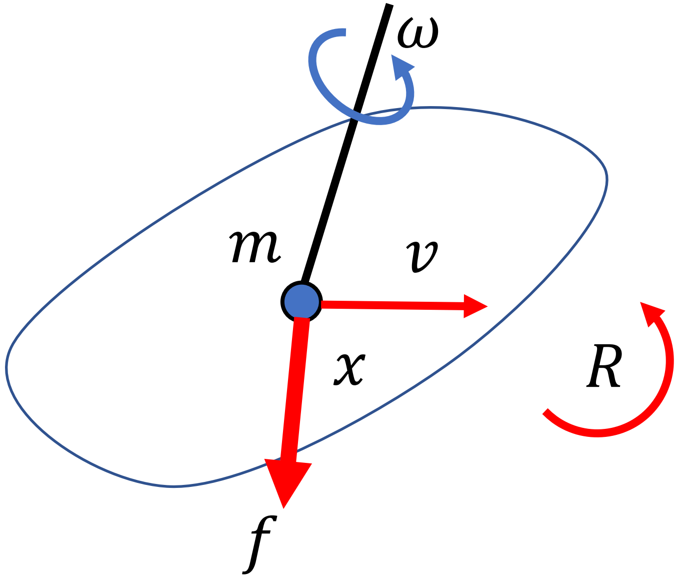
Recap: Rigid Body Dynamics



Linear	Angular
x, v	R
$p = mv$	

Newton's Second Law: $\frac{dp}{dt} = f \;\Rightarrow\; m\dot{v} = f$

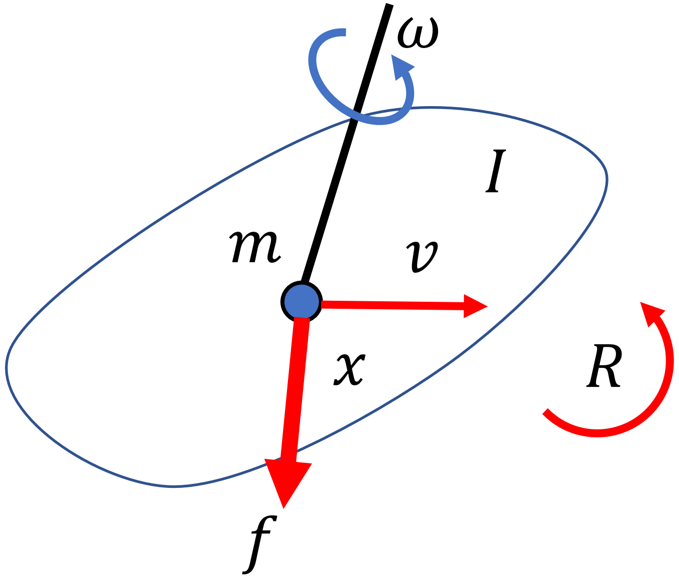
Recap: Rigid Body Dynamics



Linear	Angular
x, v	R, ω
$p = mv$	

Newton's Second Law: $\frac{dp}{dt} = f$ ➡ $m\dot{v} = f$

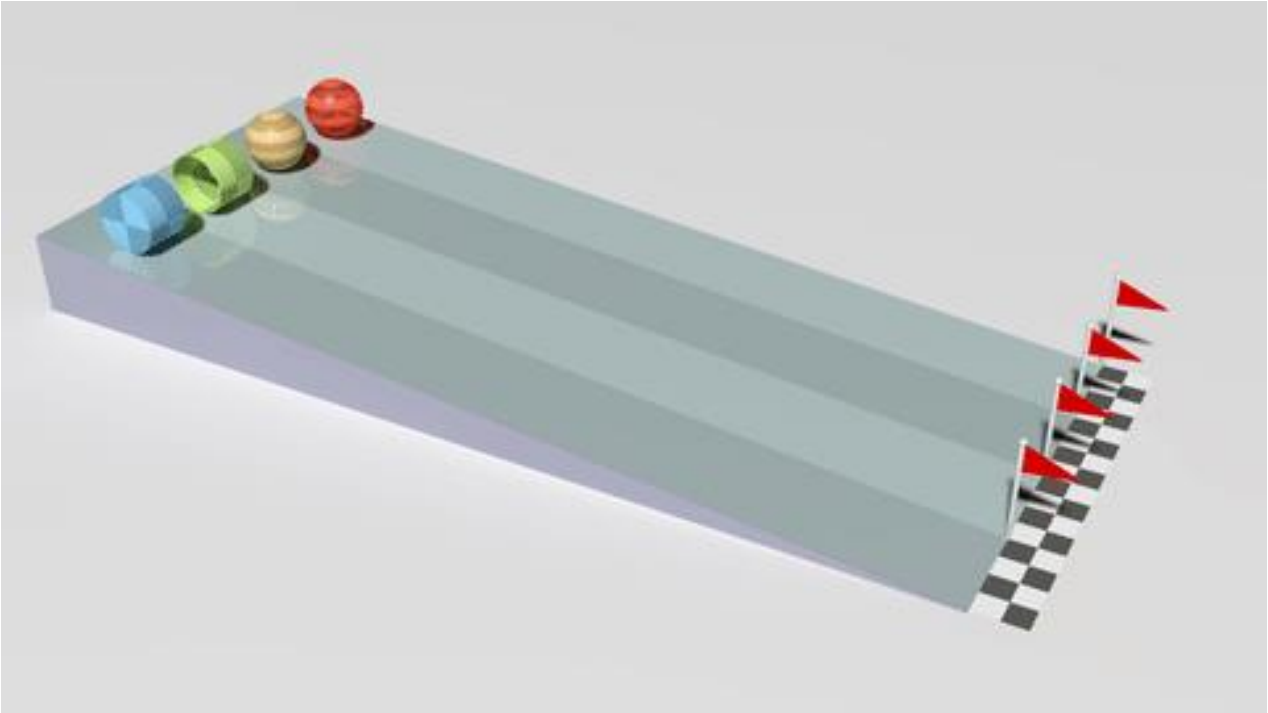
Recap: Rigid Body Dynamics



Linear	Angular
x, v	R, ω
$p = mv$	$L = I\omega$

Newton's Second Law: $\frac{dp}{dt} = f$ ➡ $m\dot{v} = f$

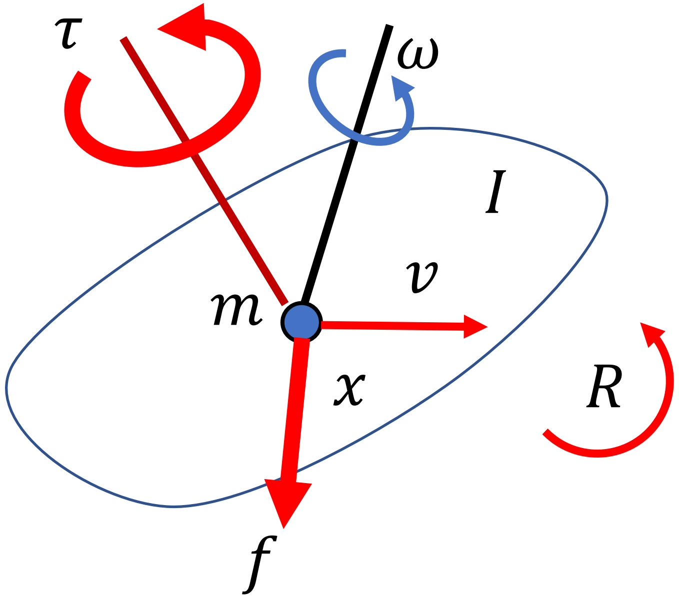
Recap: Moment of Inertia



Same mass, different shapes

Different Moments of Inertia

Recap: Rigid Body Dynamics

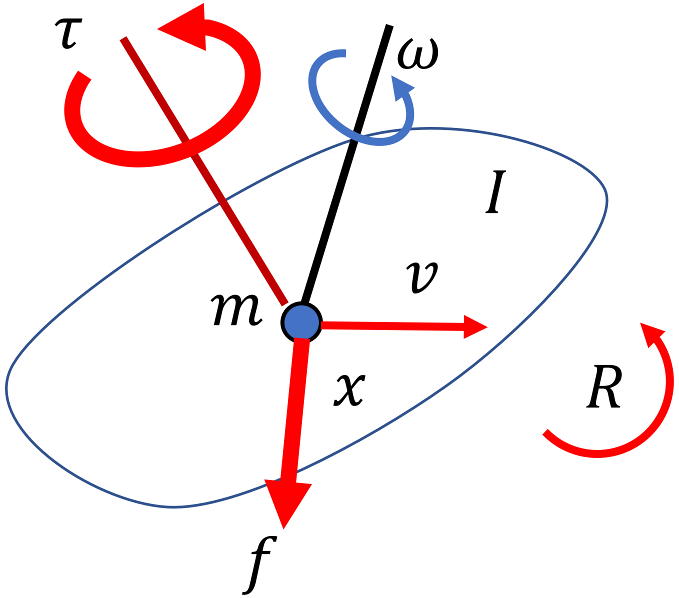


Linear	Angular
x, v	R, ω
$p = mv$	$L = I\omega$

Newton's Second Law: $\frac{dp}{dt} = f$ ➡ $m\dot{v} = f$

Euler's laws of motion: $\frac{dL}{dt} = \tau$ ➡ $I\dot{\omega} + \omega \times I\omega = \tau$

Recap: Rigid Body Dynamics



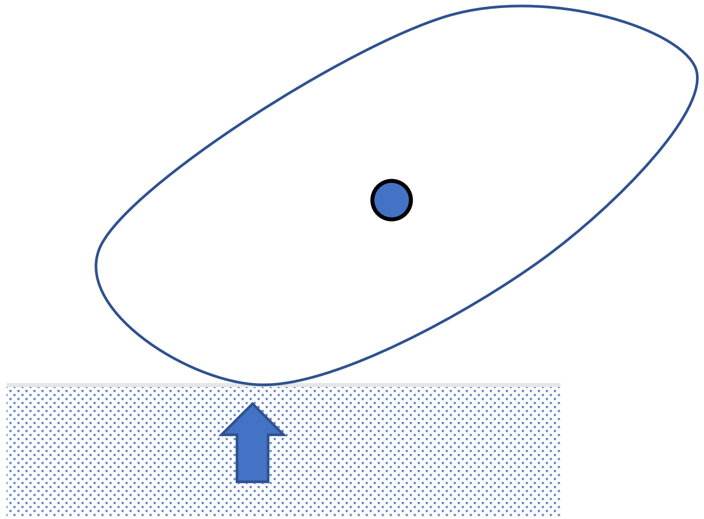
Linear	Angular
x, v	R, ω
$p = mv$	$L = I\omega$

Newton's Second Law:

Euler's laws of motion:

$$\begin{bmatrix} m\mathrm{I}_3 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \times I\omega \end{bmatrix} = \begin{bmatrix} f \\ \tau \end{bmatrix}$$

Defining a Rigid Body



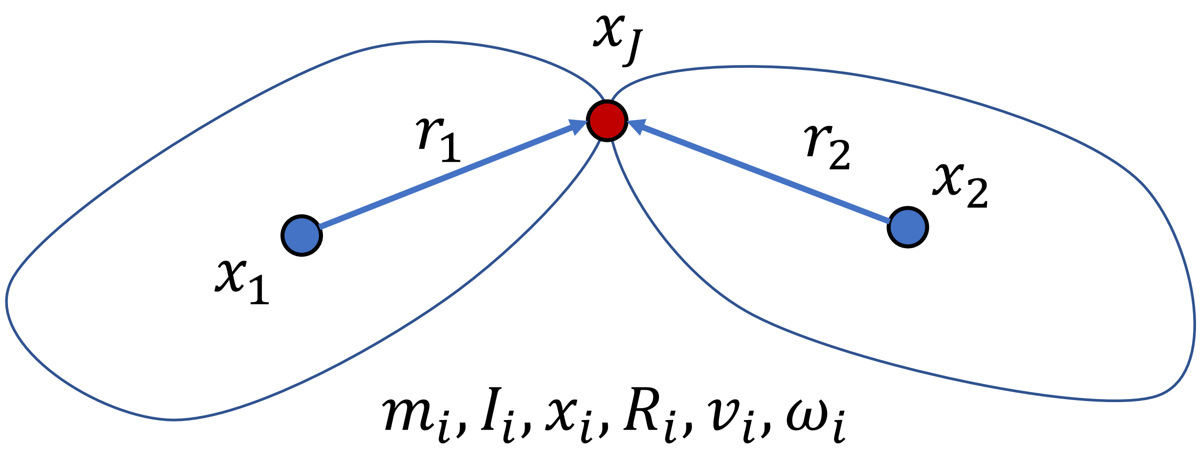
Masses: m, I

Kinematics: $\boldsymbol{x}, \boldsymbol{v}, R, \boldsymbol{\omega}$

Geometry:

- Box, Sphere, Capsule, Mesh, …
- Collision detection
- Compute m, I

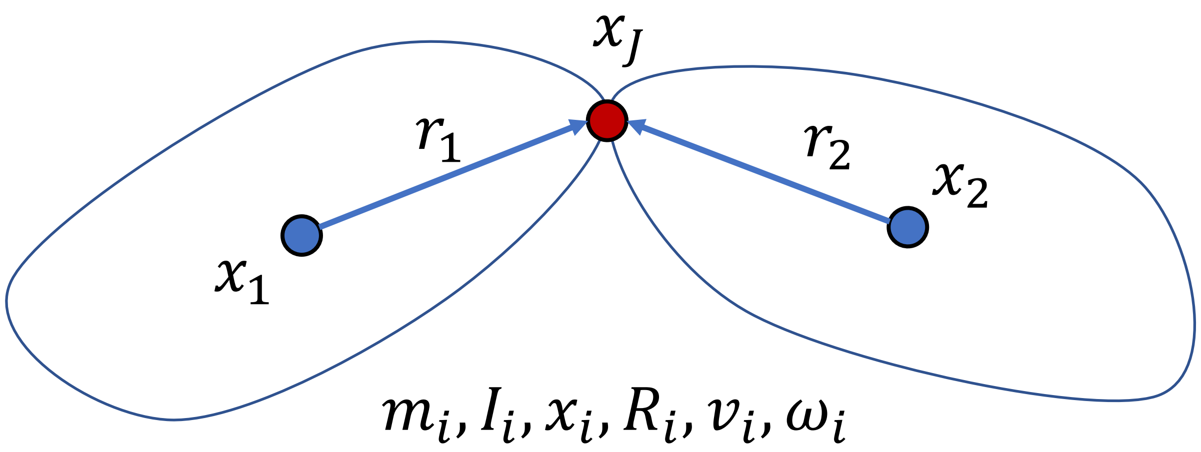
Recap: Dynamics of Articulated Rigid Bodies



$$Jv = 0$$

$$\begin{bmatrix} I_3 & -[r_1]_\times & -I_3 & [r_2]_\times \end{bmatrix} \begin{bmatrix} v_1 \\ w_1 \\ v_2 \\ w_2 \end{bmatrix} = 0$$

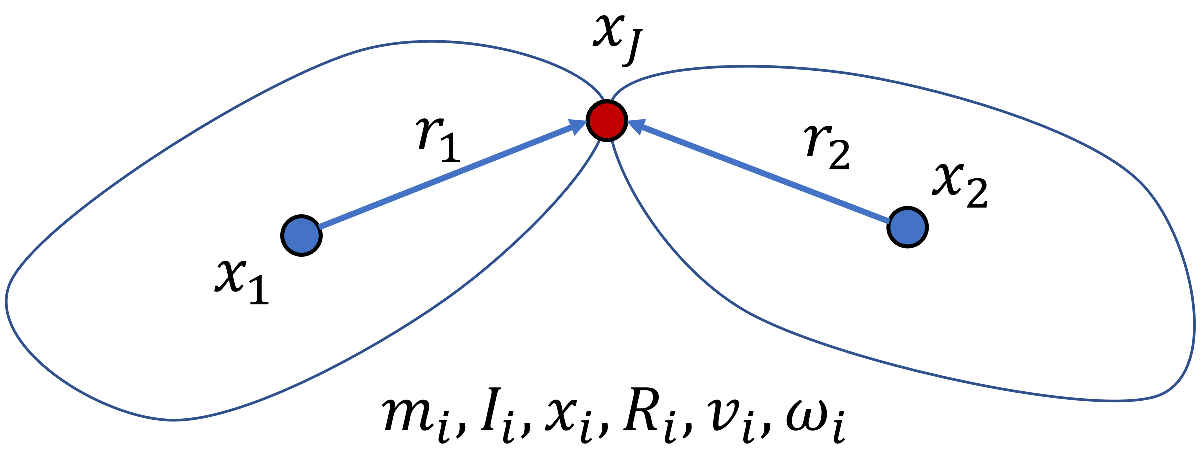
Recap: Dynamics of Articulated Rigid Bodies



$$\begin{bmatrix} m_1 \mathrm{I}_3 & & & \\ & I_1 & & \\ & & m_2 \mathrm{I}_3 & \\ & & & I_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{\omega}_1 \\ \dot{v}_2 \\ \dot{\omega}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_1 \times I_1 \omega_1 \\ 0 \\ \omega_2 \times I_2 \omega_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ \tau_1 \\ f_2 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} I_3 \\ [r_1]_\times \\ -I_3 \\ -[r_2]_\times \end{bmatrix} \lambda$$

$$Jv = 0$$

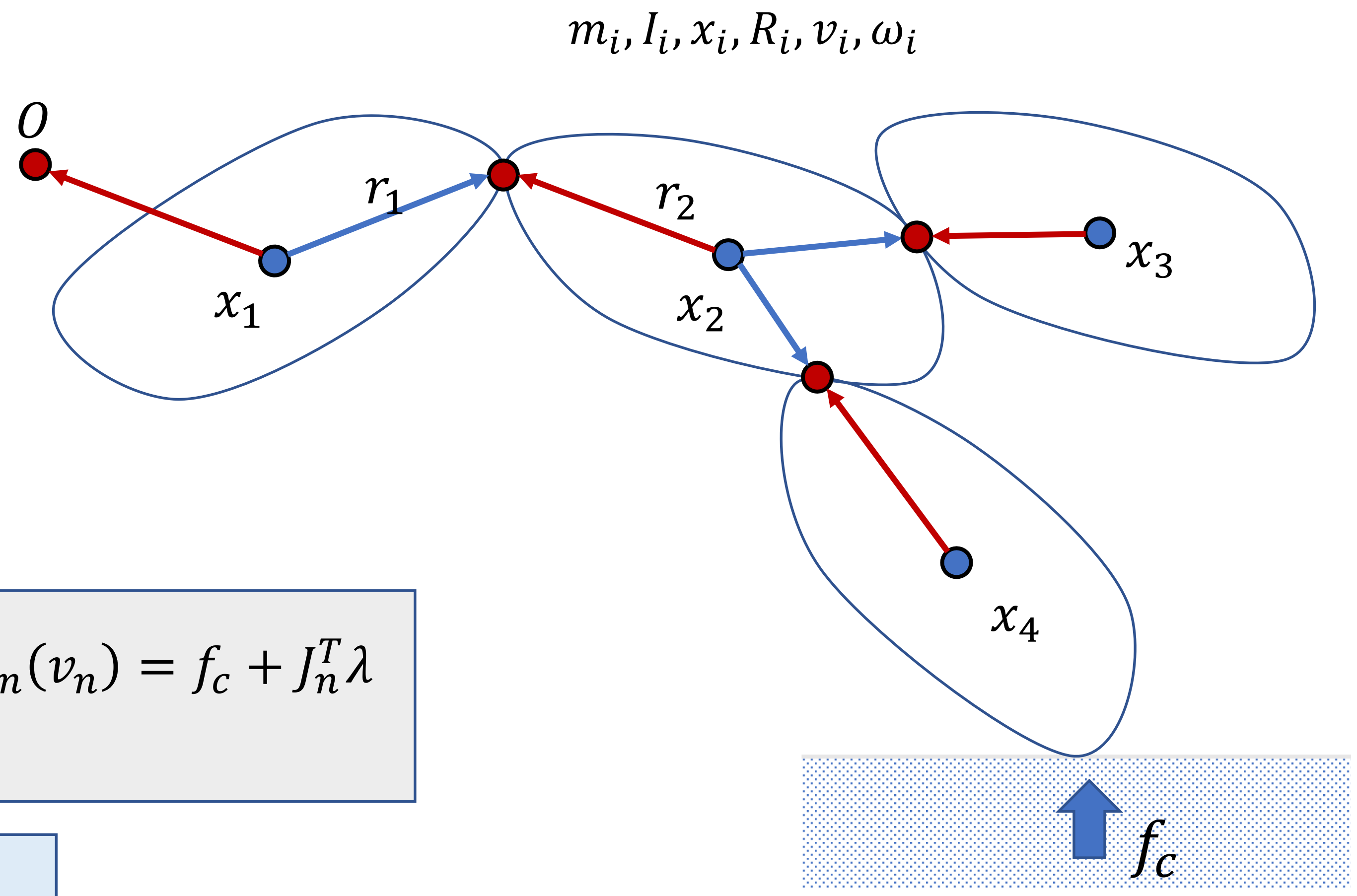
Recap: Dynamics of Articulated Rigid Bodies



$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{J}^{\boldsymbol{T}}\boldsymbol{\lambda}$$

$$Jv = 0$$

Recap: Simulation of a Rigid Body System



$$I_n = R_n I_0 R_n^T$$

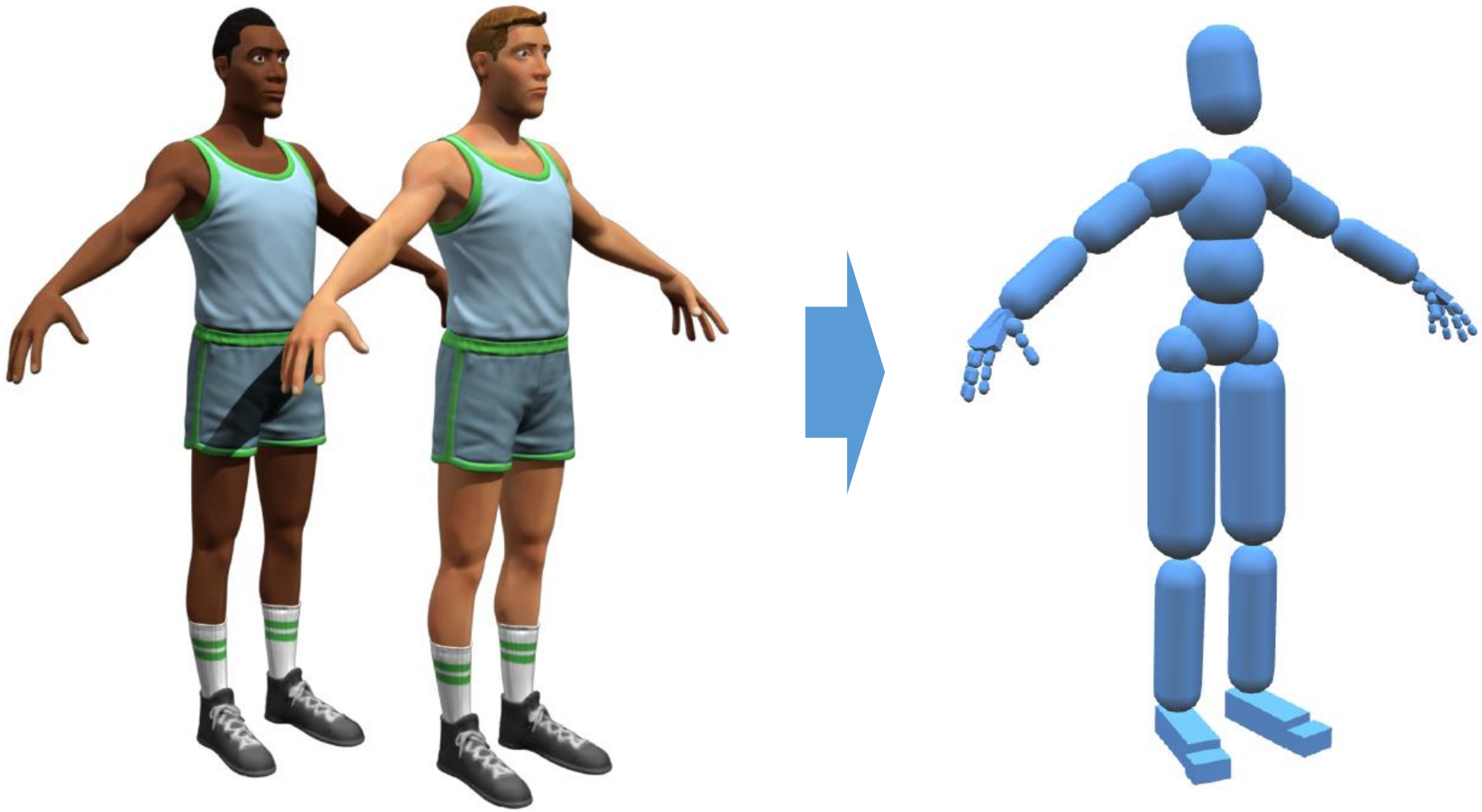
$$M_n(v_{n+1} - v_n)/_h + C_n(v_n) = f_c + J_n^T \lambda$$

$$J_n v_{n+1} = c_n$$

$$x_{n+1} = x_n + h v_{n+1}$$

$$q_{n+1} = q_n + \frac{h}{2} \bar{\omega}_{n+1} q$$

Defining a Simulated Character



[Liu et al 2018]

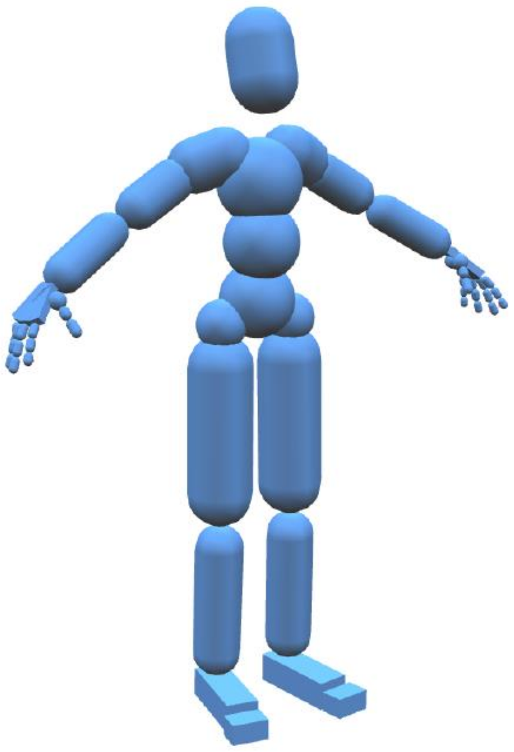
Rigid bodies:

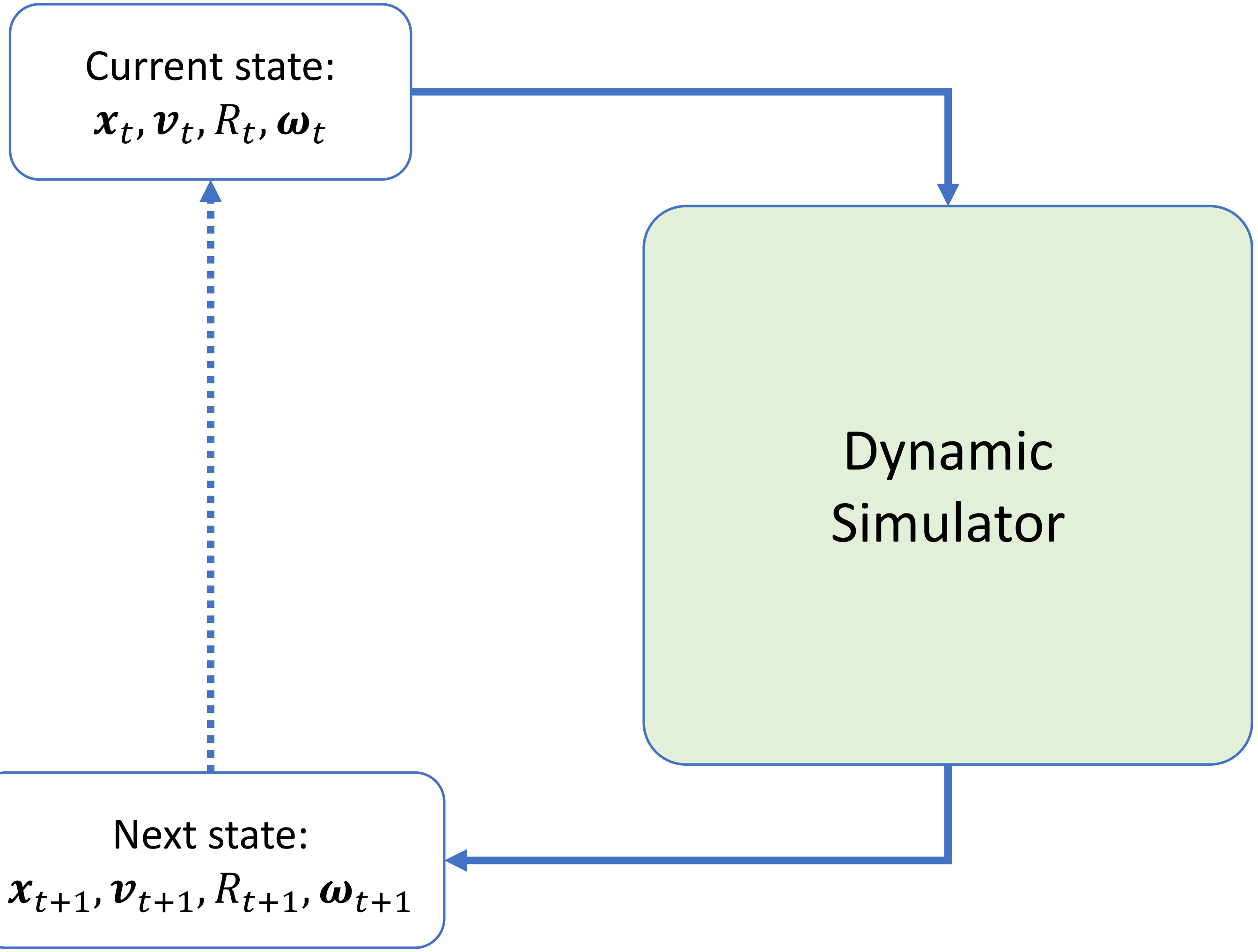
- $m_i, I_i, \boldsymbol{x}_i, R_i$
- Geometries

Joints:

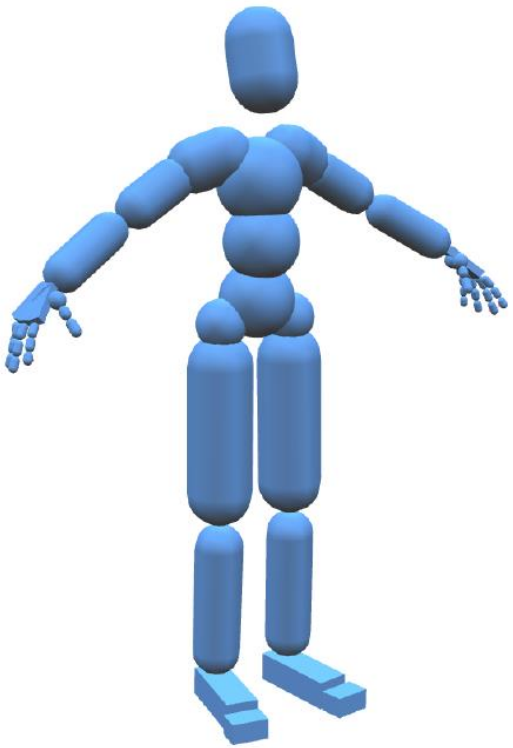
- Position
- Type
- Bodies

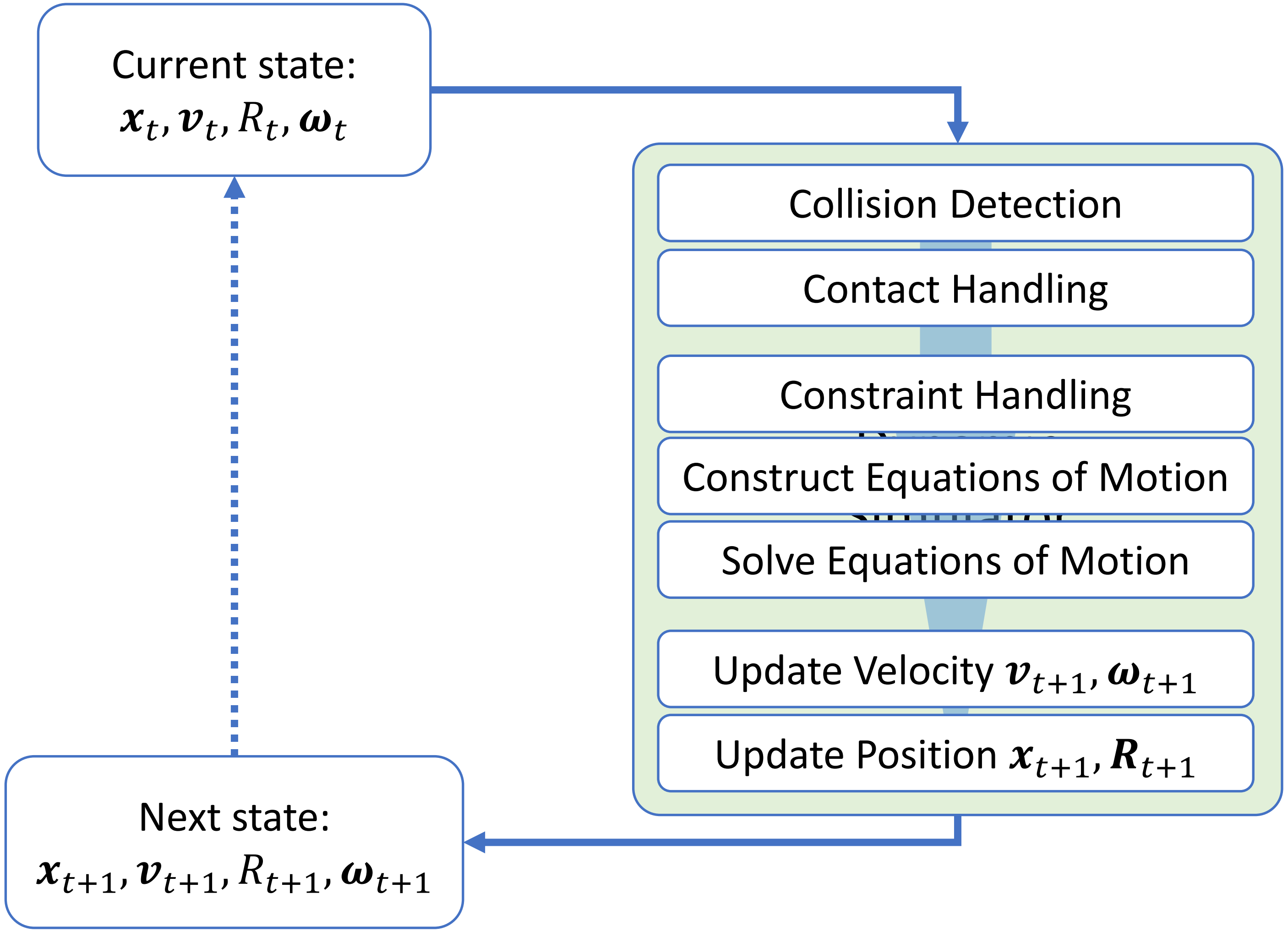
Simulating a Character



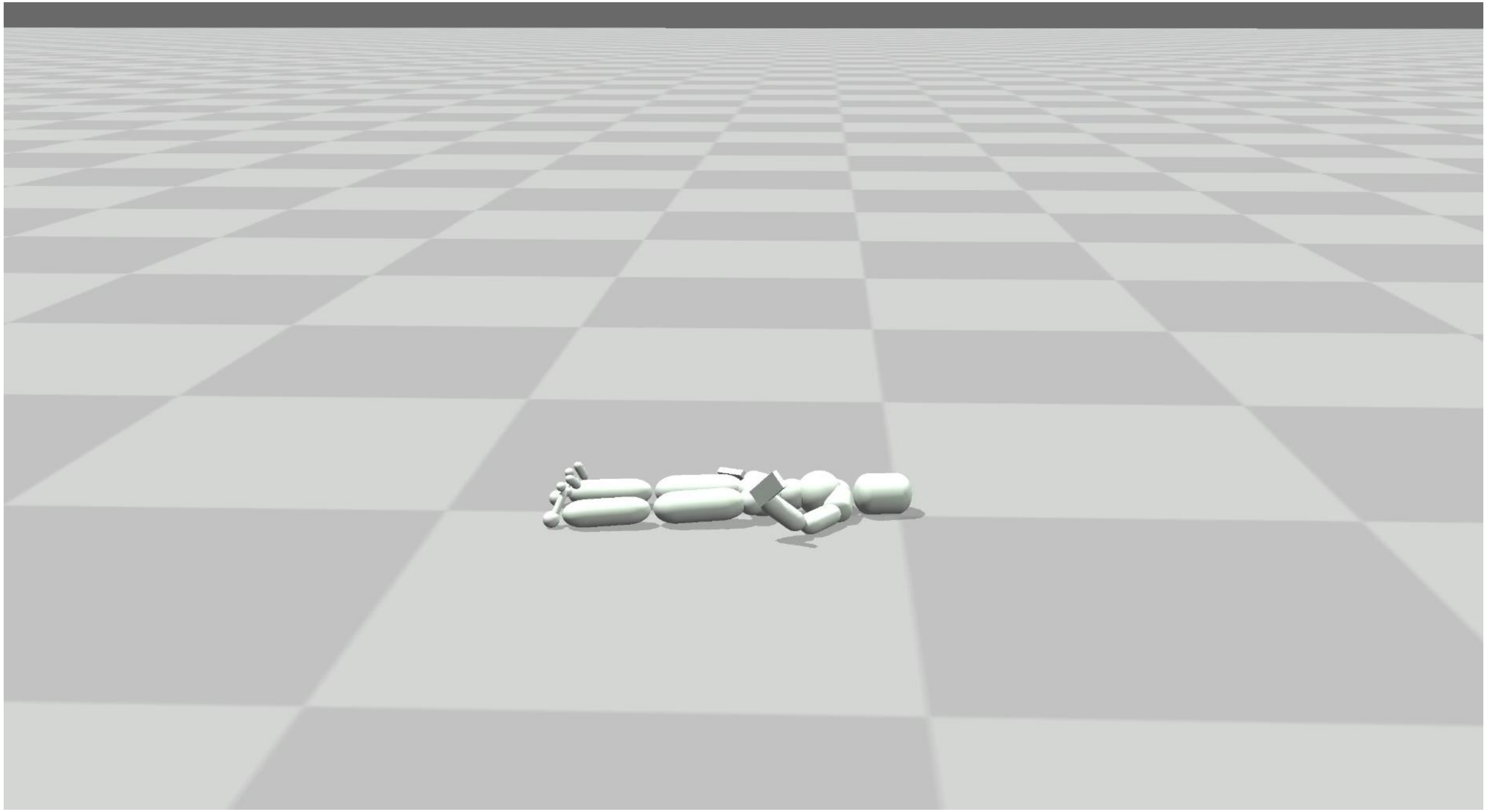


Simulating a Character

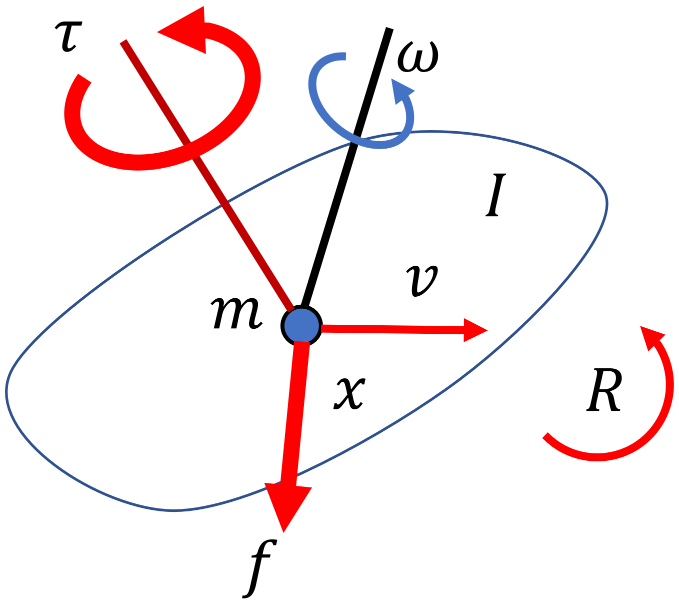




Ragdoll Simulation



Actuating a Rigid Body



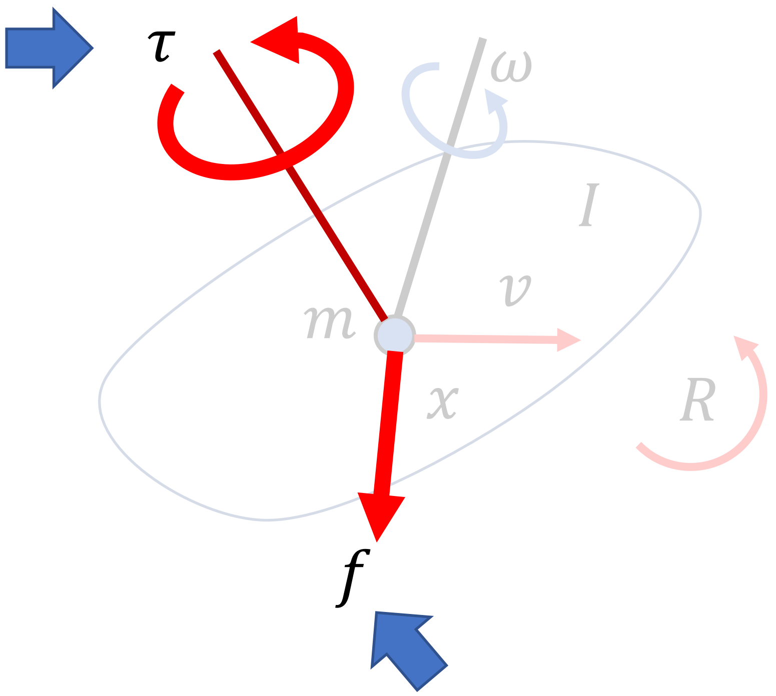
Linear	Angular
x, v	R, ω
$p = mv$	$L = I\omega$

Newton's Second Law:

Euler's laws of motion:

$$\begin{bmatrix} m\mathrm{I}_3 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \times I\omega \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{\tau} \end{bmatrix}$$

Actuating a Rigid Body



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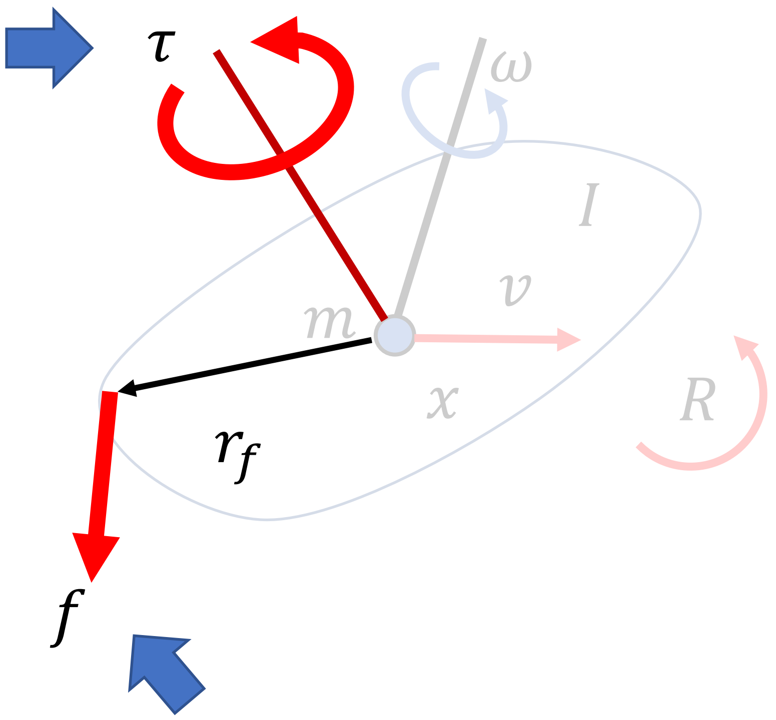
Actuating a Rigid Body



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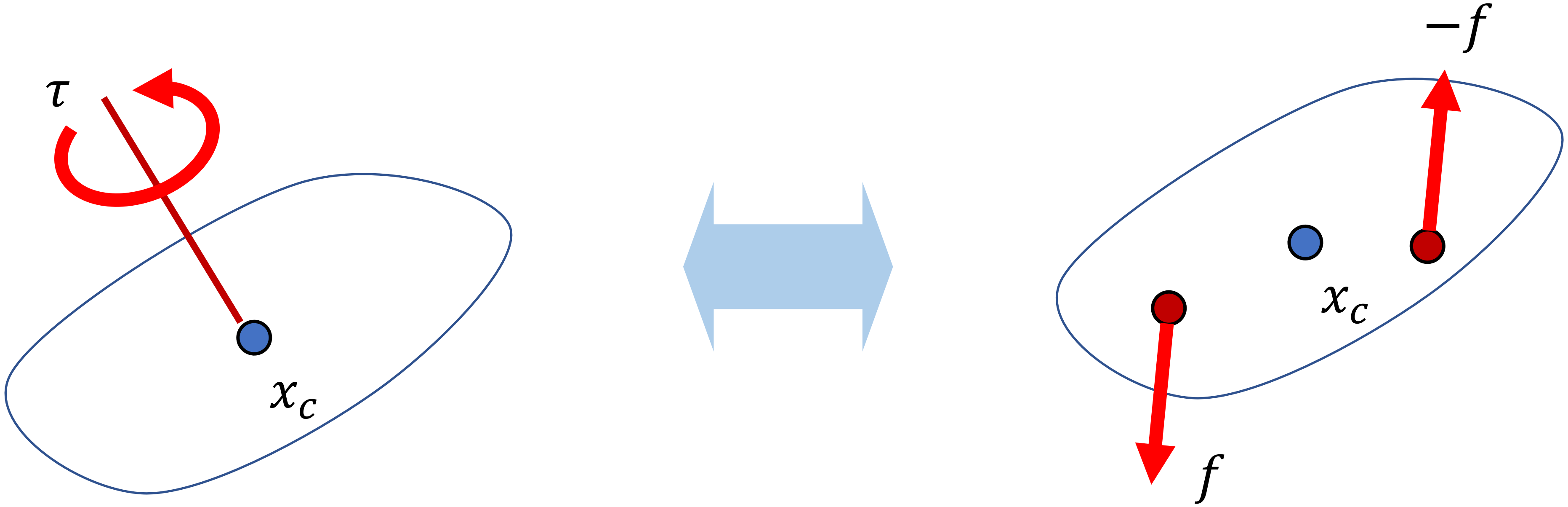
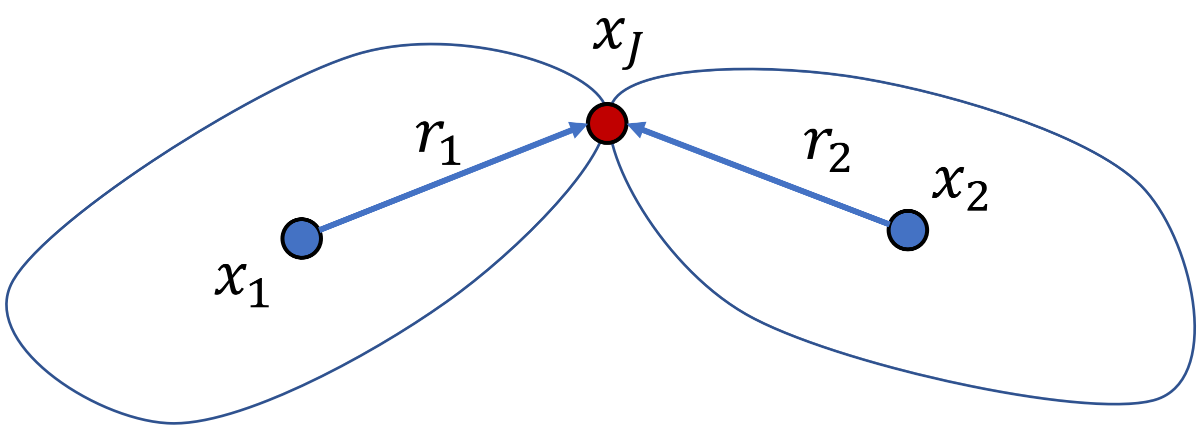
Actuating a Rigid Body



Actuating Articulated Rigid Bodies

$m_i, I_i, x_i, R_i, v_i, \omega_i$

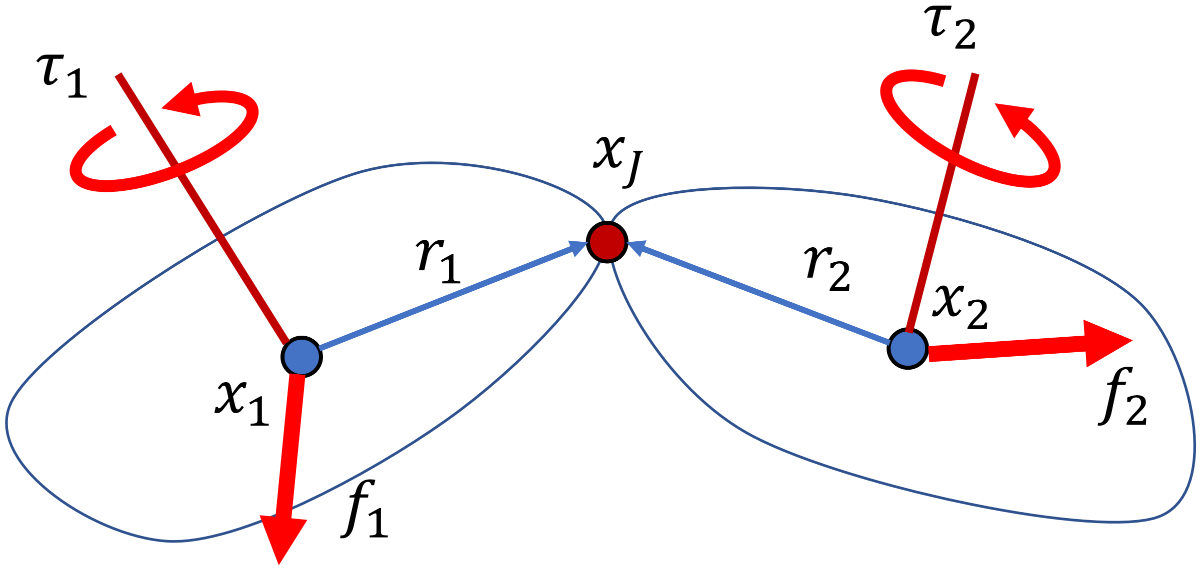


$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{J}^T\boldsymbol{\lambda}$$

$$Jv = 0$$

Actuating Articulated Rigid Bodies

$m_i, I_i, x_i, R_i, v_i, \omega_i$



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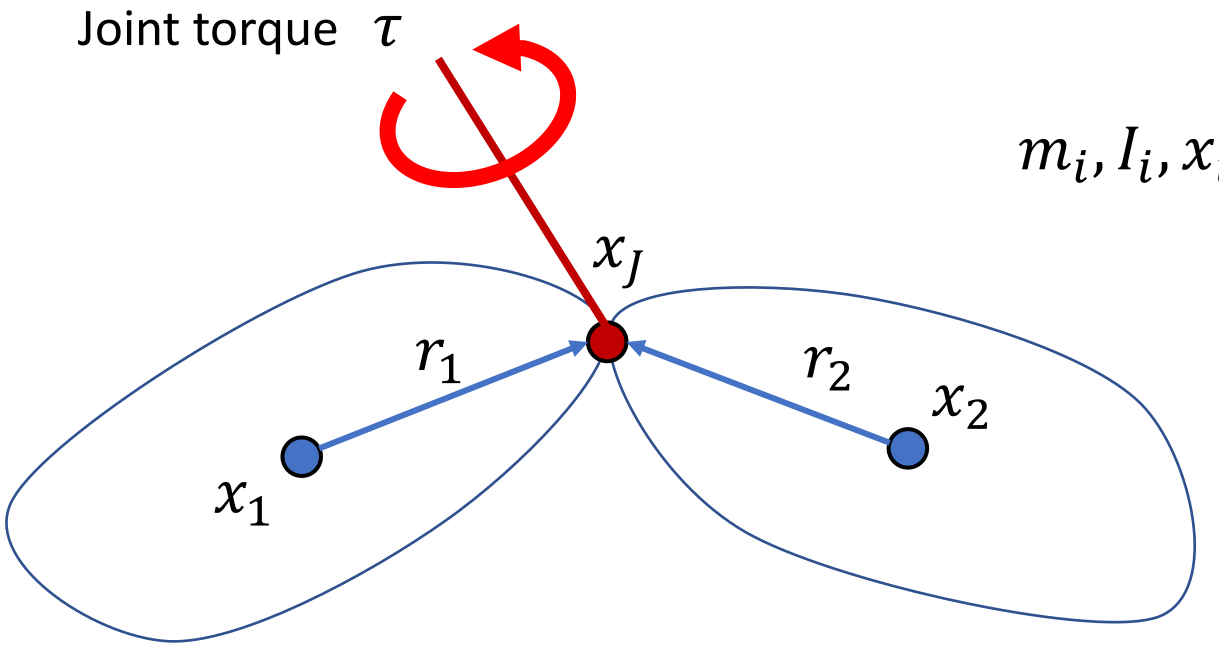
Actuating Articulated Rigid Bodies



$m_i, I_i, x_i, R_i, v_i, \omega_i$

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{J}^T\boldsymbol{\lambda}$$

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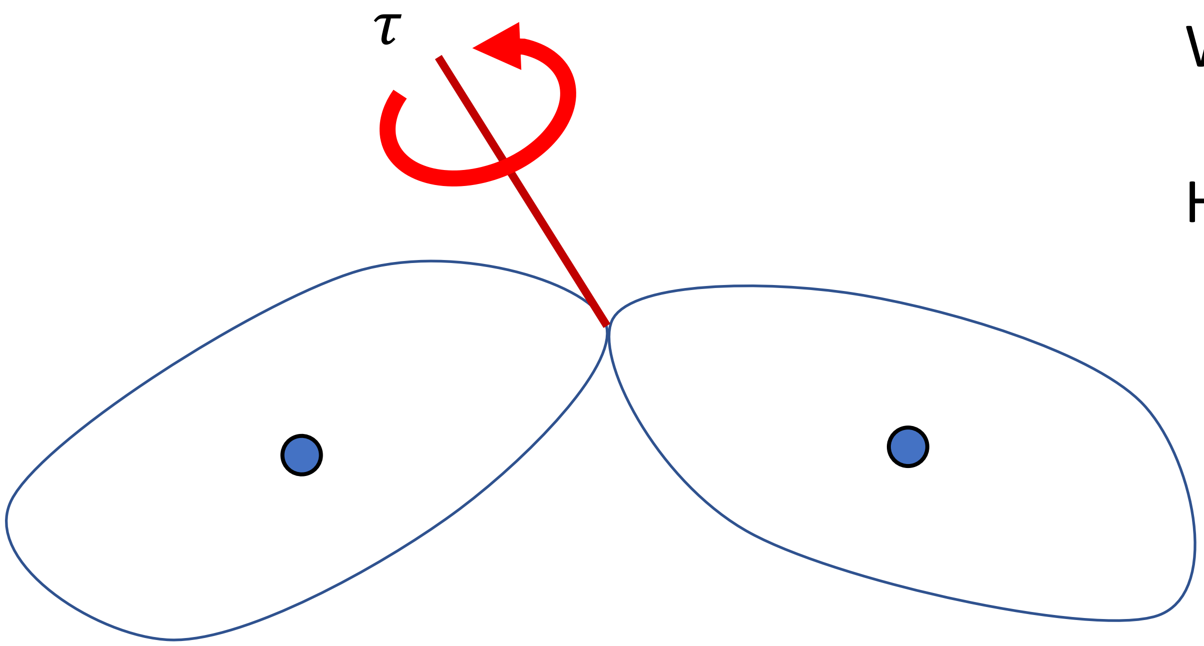
Joint Torques



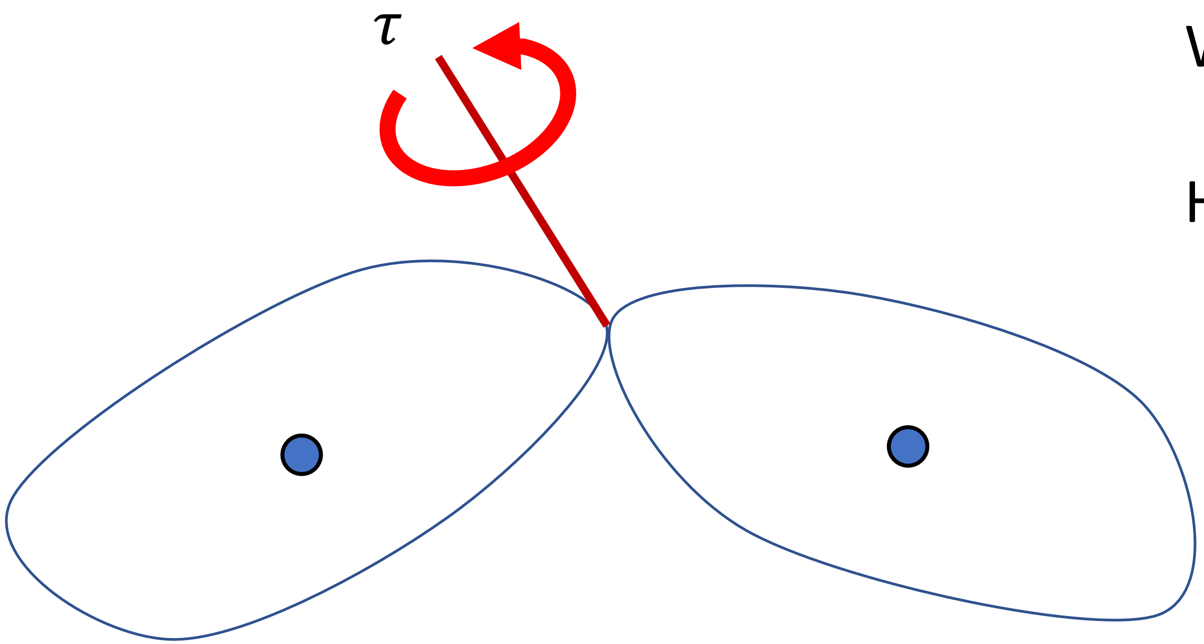
What is a joint torque?

How is a joint torque applied?

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + \boldsymbol{J}^T\boldsymbol{\lambda}$$

$$Jv = 0$$

Joint Torques



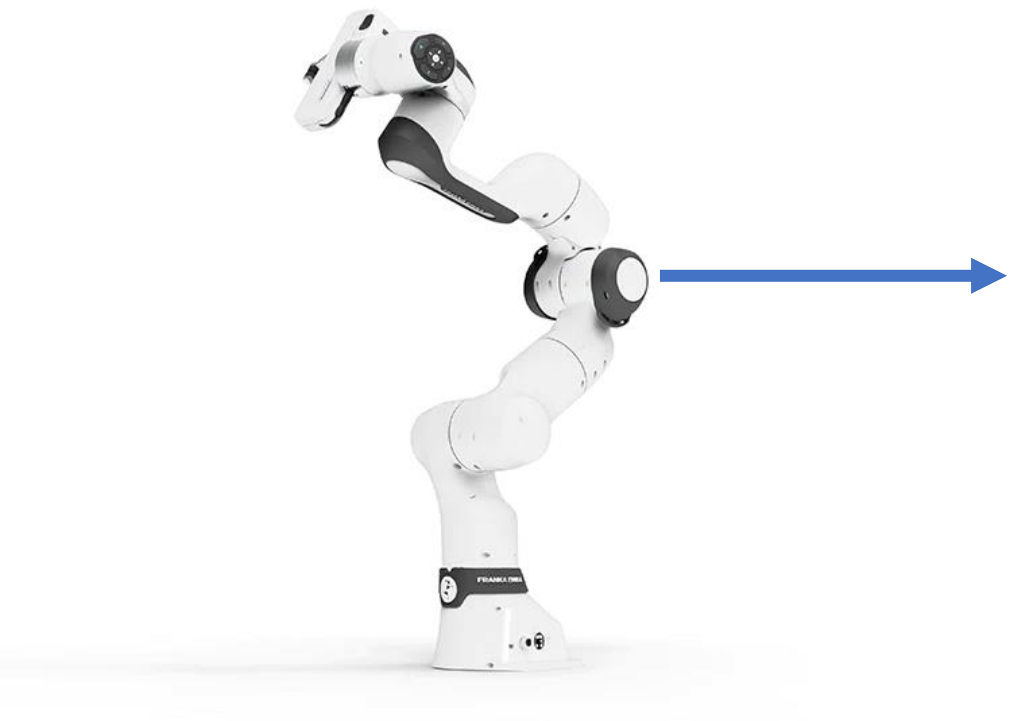
What is a joint torque?

How is a joint torque applied?

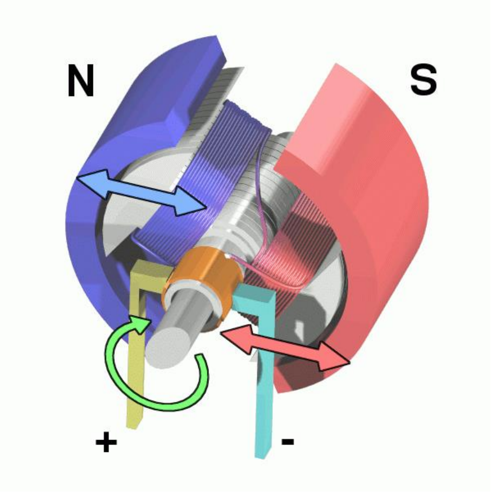
$$\begin{bmatrix} m_1 \mathrm{I}_3 & & & \\ & I_1 & & \\ & & m_2 \mathrm{I}_3 & \\ & & & I_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{\omega}_1 \\ \dot{v}_2 \\ \dot{\omega}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_1 \times I_1 \omega_1 \\ 0 \\ \omega_2 \times I_2 \omega_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ \tau_1 \\ f_2 \\ \tau_2 \end{bmatrix} + J^T \lambda$$

$$Jv = 0$$

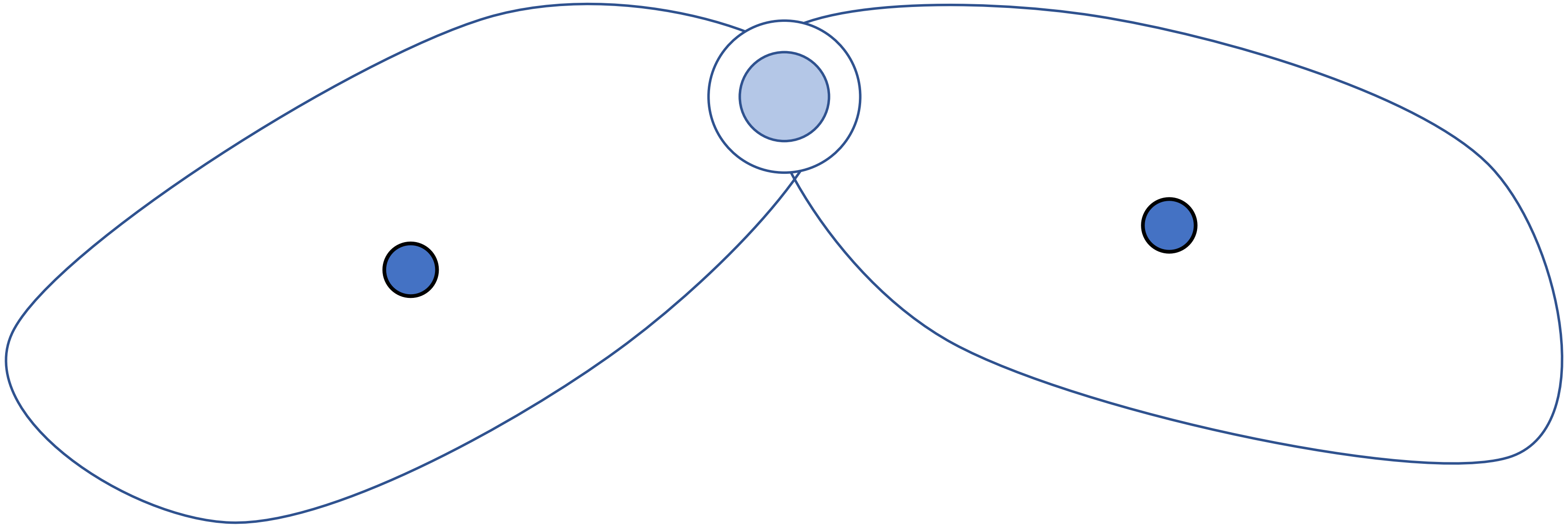
Joint Torques



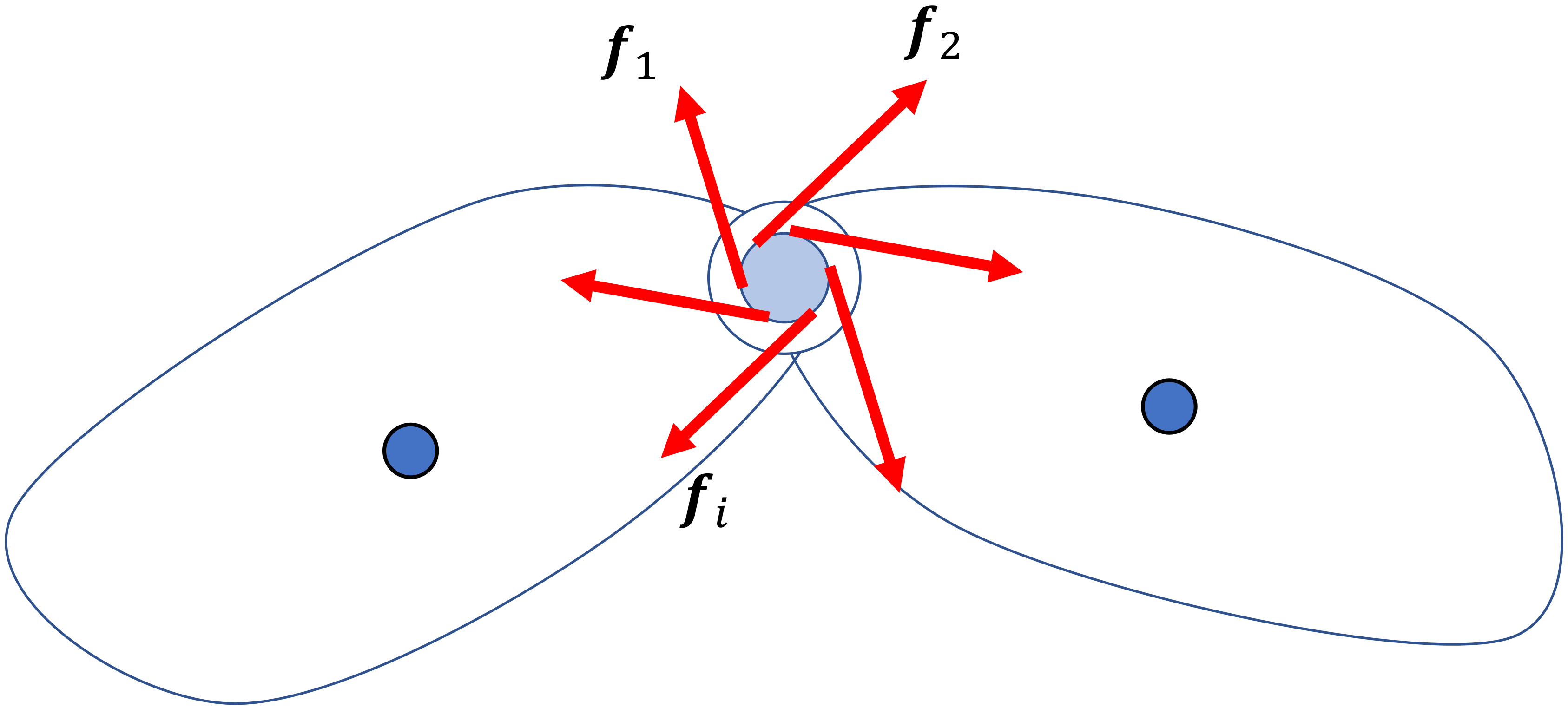
Franka



Joint Torques



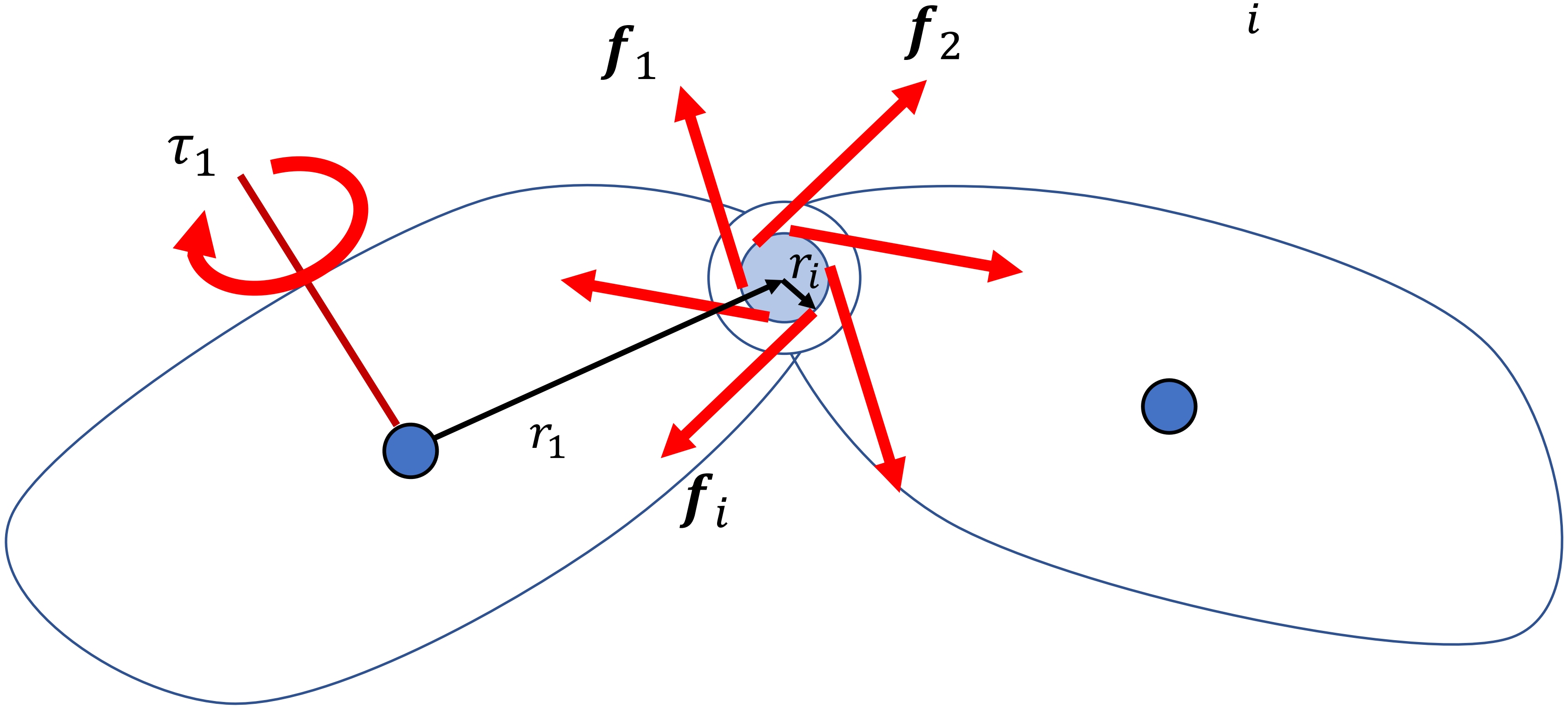
Joint Torques



$$\sum_i f_i = 0$$

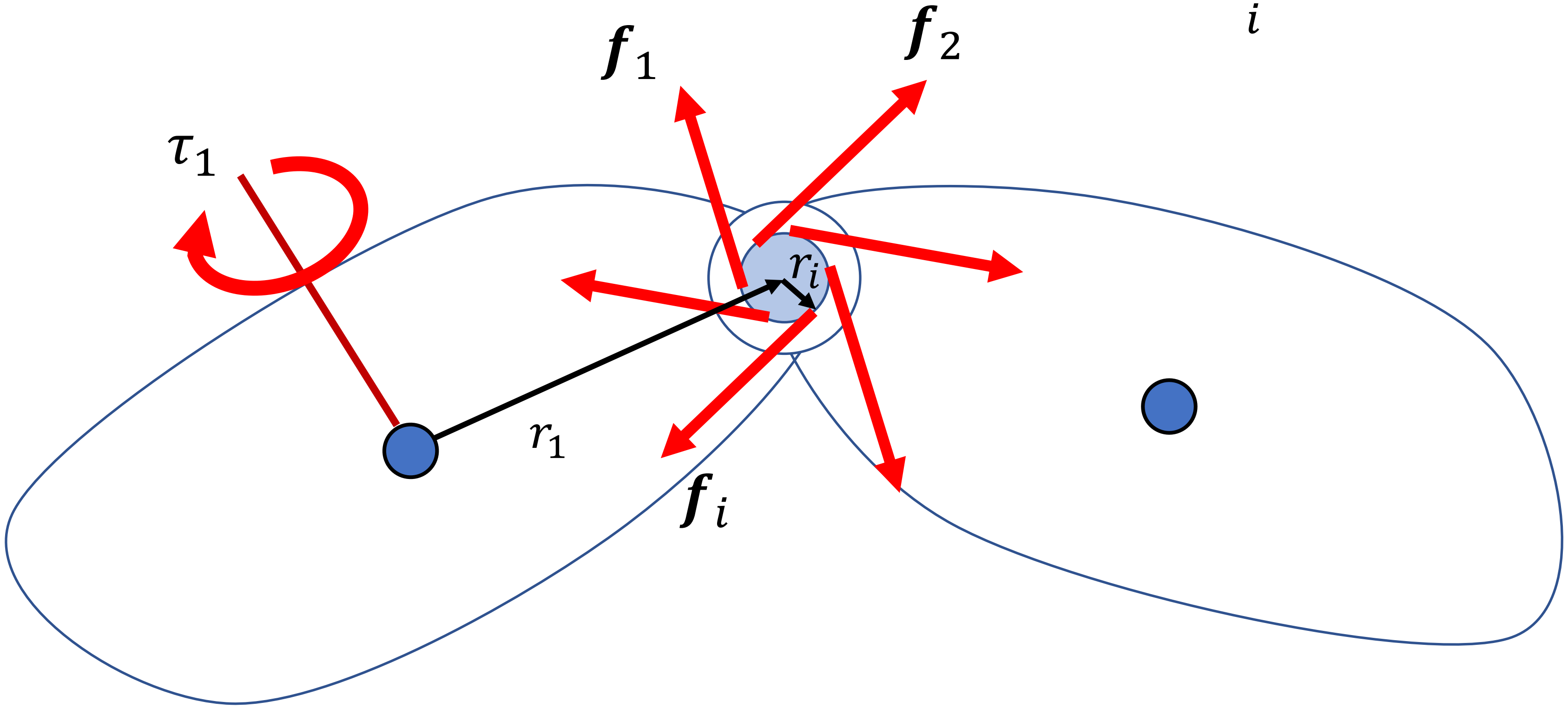
Joint Torques

$$\sum_i f_i = 0$$



$$\tau_1 = \sum_i (r_1 + r_i) \times f_i = r_1 \times \sum_i f_i + \sum_i r_i \times f_i$$

Joint Torques

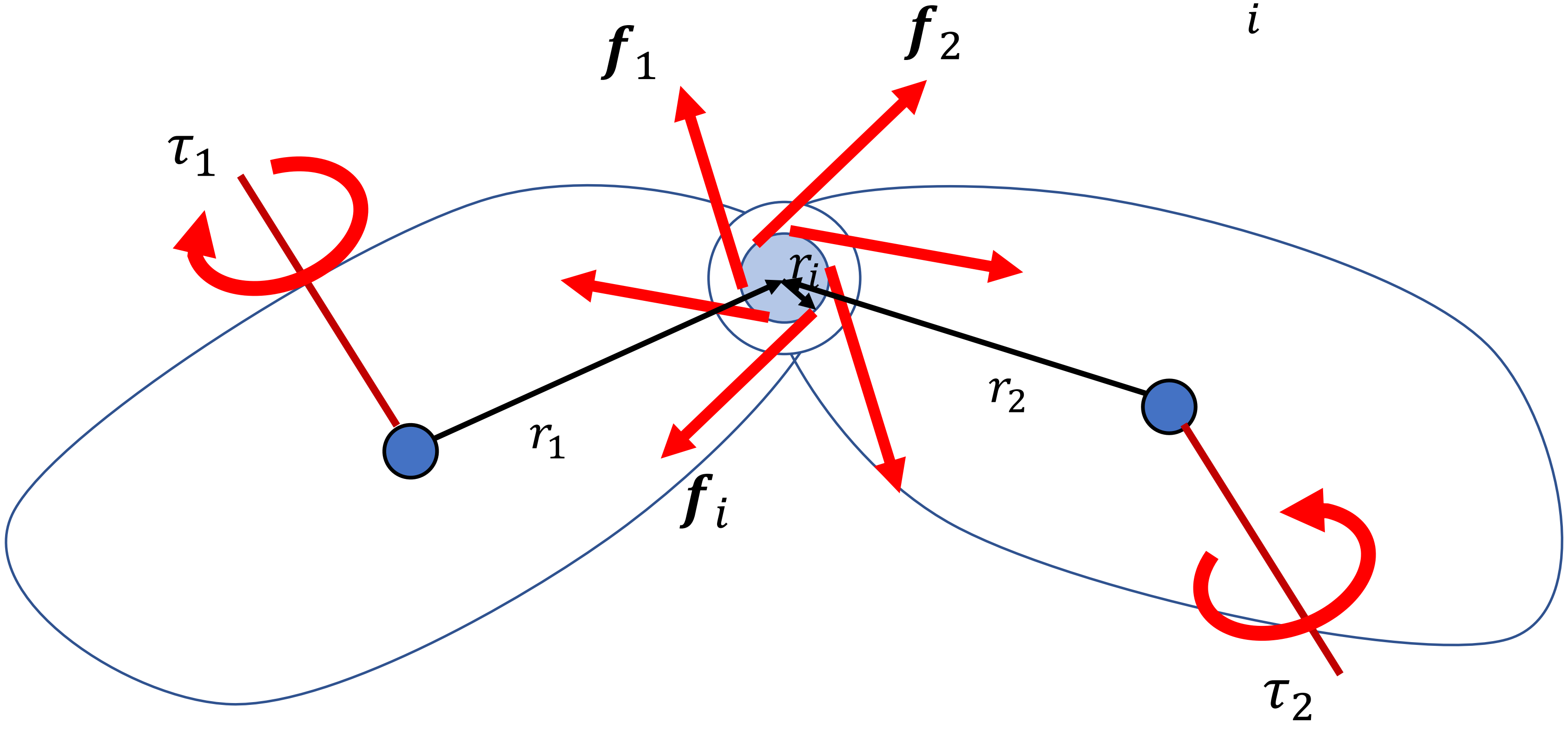


$$\sum_i f_i = 0$$

$$\tau_1 = \sum_i r_i \times f_i$$

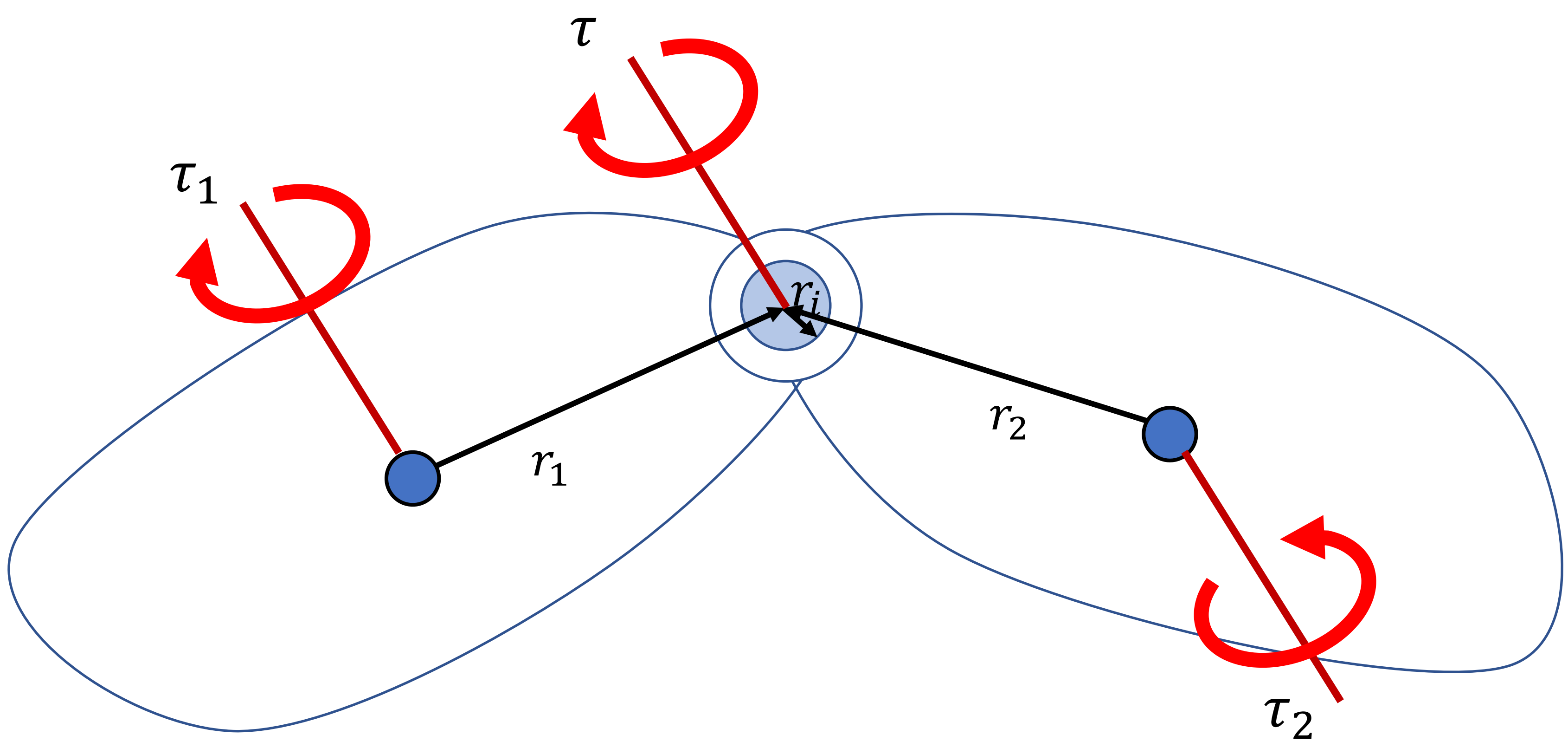
Joint Torques

$$\sum_i f_i = 0$$



$$\tau_1 = \sum_i r_i \times f_i \qquad \tau_2 = -\sum_i r_i \times f_i$$

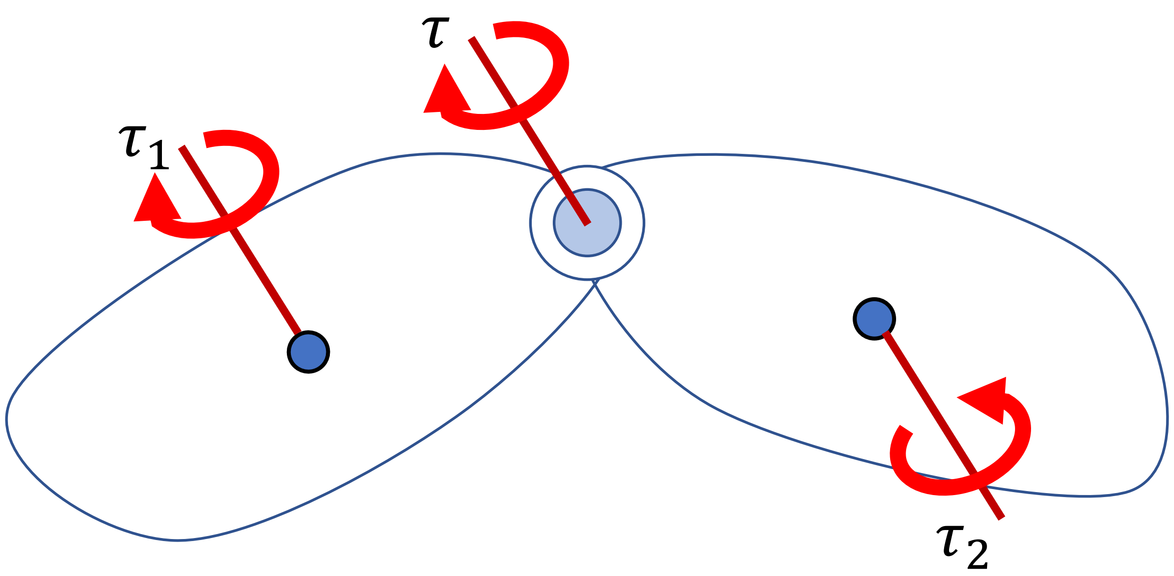
Joint Torques



$\tau_1 = \tau$

$\tau_2 = -\tau$

Joint Torques



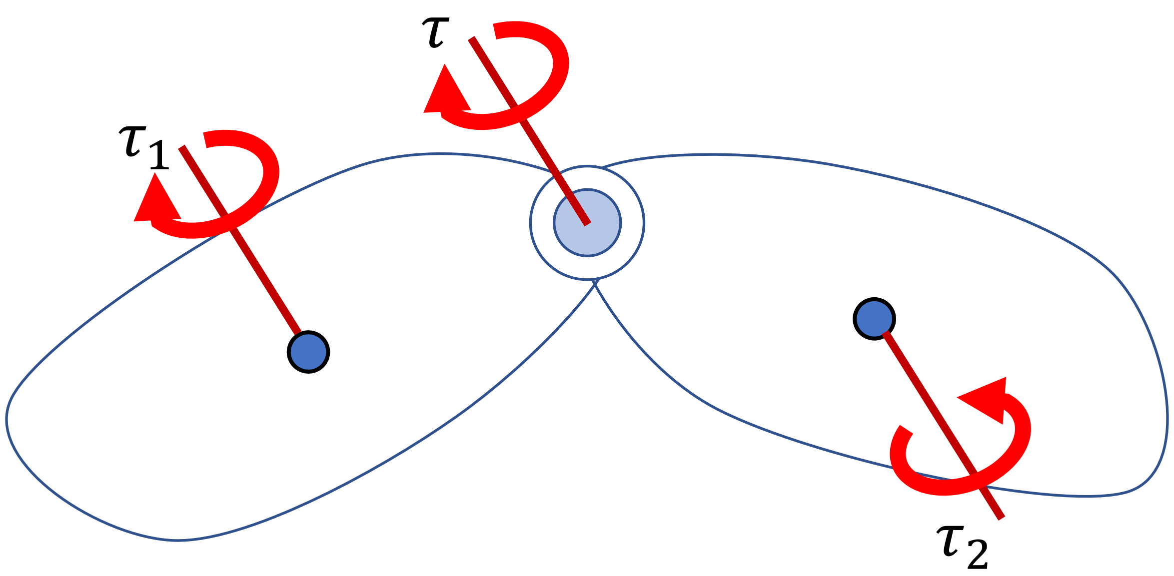
$\tau_1 = \tau$

$\tau_2 = -\tau$

Applying a joint torque τ:

- Add τ to one attached body
- Add $-\tau$ to the other attached body

Joint Torques



$\tau_1 = \tau$

$\tau_2 = -\tau$

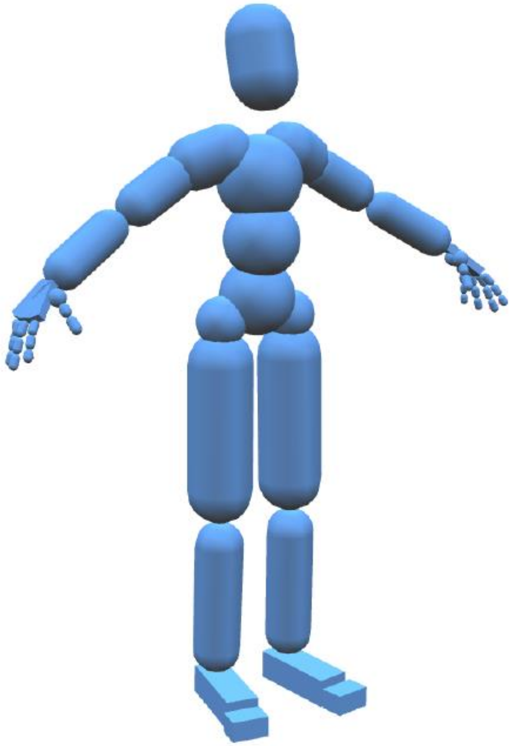
Applying a joint torque τ:

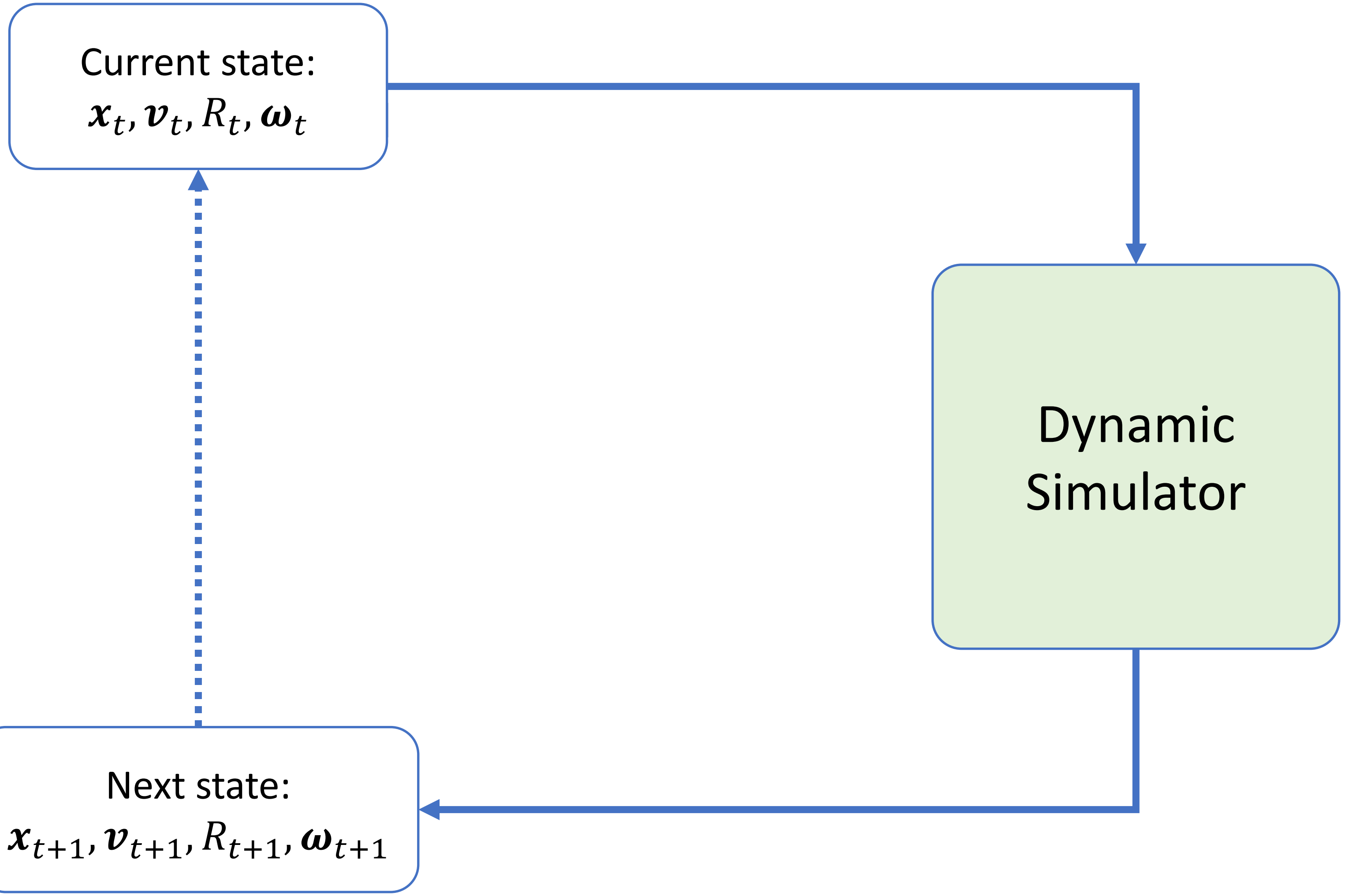
- Add τ to one attached body
- Add $-\tau$ to the other attached body

$$M\begin{bmatrix}\dot{v}_1\\ \dot{\omega}_1\\ \dot{v}_2\\ \dot{\omega}_2\end{bmatrix} + \begin{bmatrix}0\\ \omega_1 \times I_1\omega_1\\ 0\\ \omega_2 \times I_2\omega_2\end{bmatrix} = \begin{bmatrix}0\\ \tau\\ 0\\ -\tau\end{bmatrix} + J^T\lambda$$

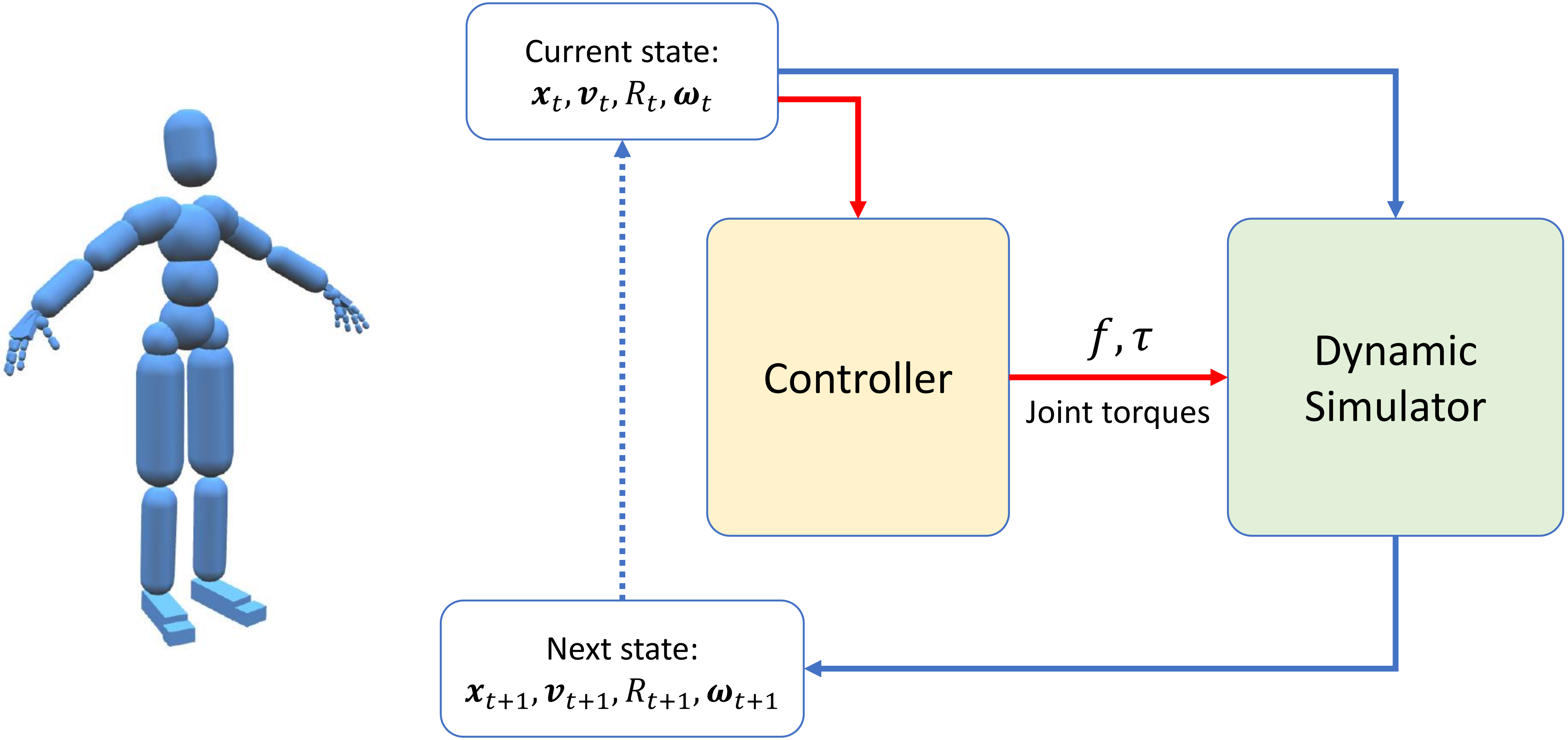
$$Jv = 0$$

Simulating a Character





Simulating + Controlling a Character



Forward Dynamics vs. Inverse Dynamics

Equations of motion of the system:

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + J^T\lambda \qquad Jv = 0$$

Forward Dynamics vs. Inverse Dynamics

Equations of motion of the system:

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + J^T\lambda \qquad Jv = 0$$

Forward dynamics: $[\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{f}] \mapsto \dot{\boldsymbol{v}}$

given a set of force/torques, compute the motion

Forward Dynamics vs. Inverse Dynamics

Equations of motion of the system:

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + J^T\lambda \qquad Jv = 0$$

Forward dynamics: $[\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{f}] \mapsto \dot{\boldsymbol{v}}$

given a set of force/torques, compute the motion

Inverse dynamics: $[\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{v}}] \mapsto \boldsymbol{f}$

given a motion, compute the forces/torques that give rise to it

Forward Dynamics vs. Inverse Dynamics

Equations of motion of the system:

$$M\dot{\boldsymbol{v}} + C(\boldsymbol{x}, \boldsymbol{v}) = \boldsymbol{f} + J^T\lambda \qquad Jv = 0$$

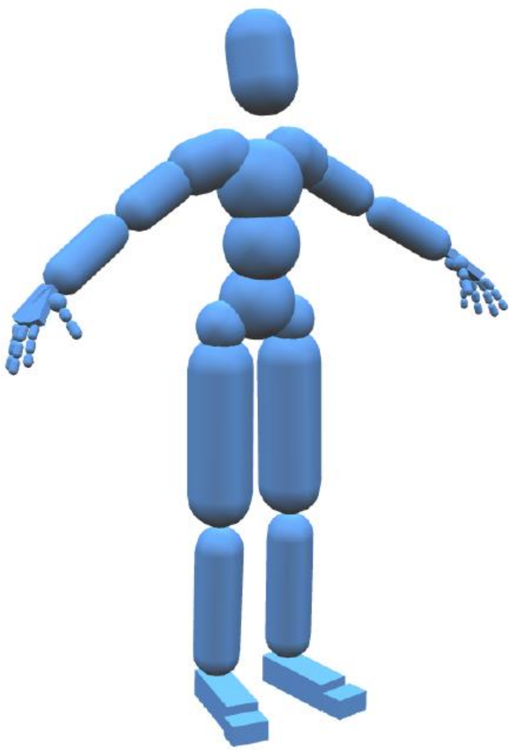
Forward dynamics: $[\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{f}] \mapsto \dot{\boldsymbol{v}}$

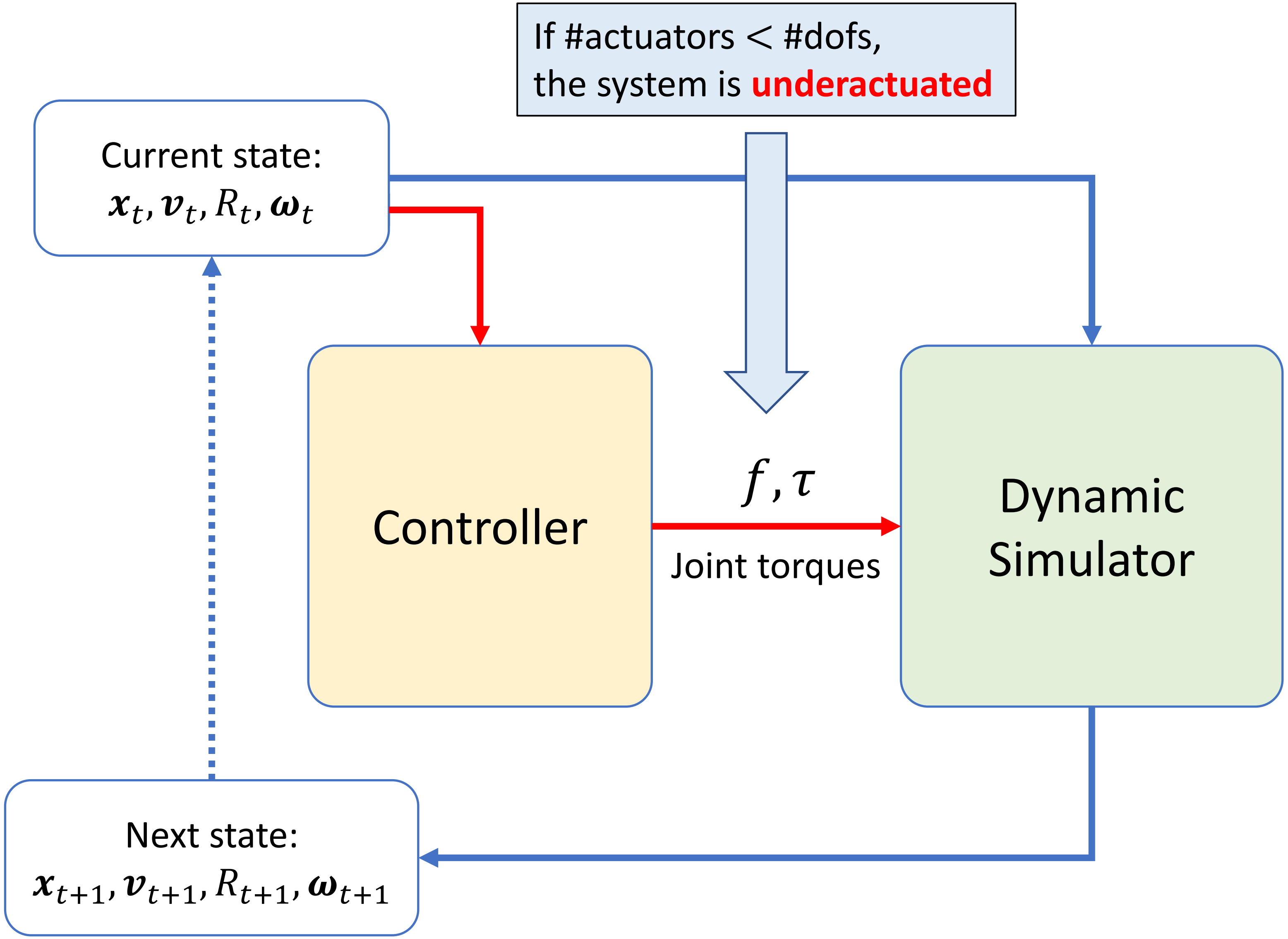
dynamic simulator — given a set of force/torques, compute the motion

Inverse dynamics: $[\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{v}}] \mapsto \boldsymbol{f}$

controller — given a motion, compute the forces/torques that give rise to it

Simulating + Control



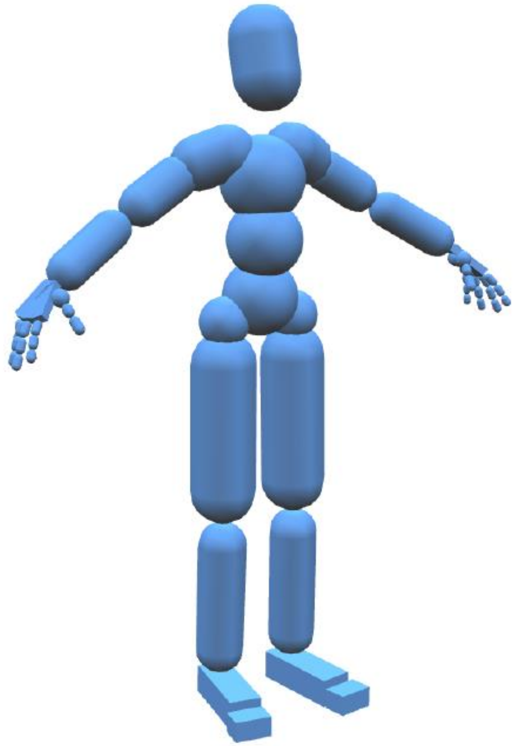


Fully-Actuated vs. Underactuated

Fully-Actuated

If #actuators $\geq$ #dofs, the system is **fully-actuated**

Underactuated



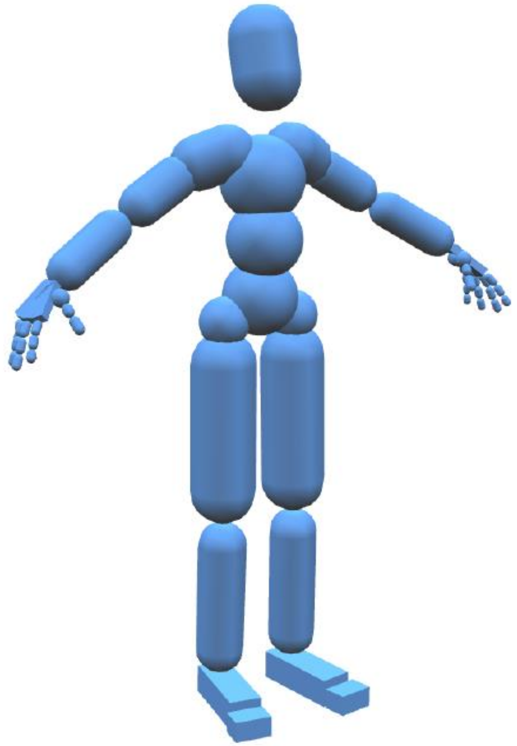
If #actuators $<$ #dofs, the system is **underactuated**

Fully-Actuated vs. Underactuated

Fully-Actuated

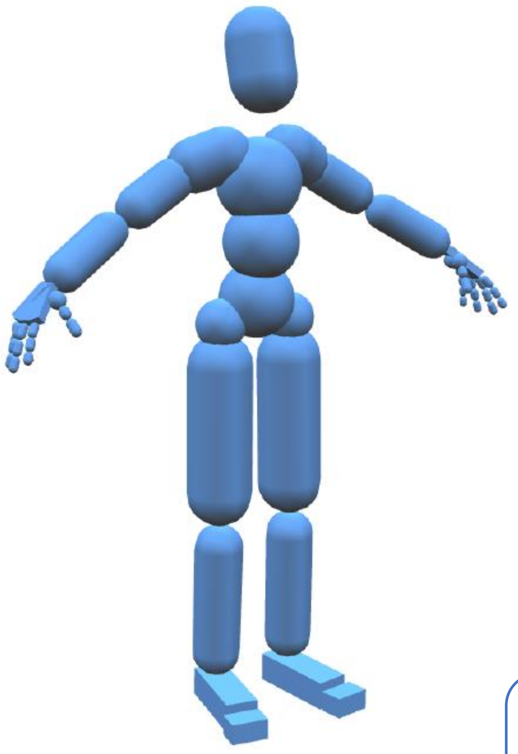
For any $[\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{v}}]$, there exists an $\boldsymbol{f}$ that produces the motion

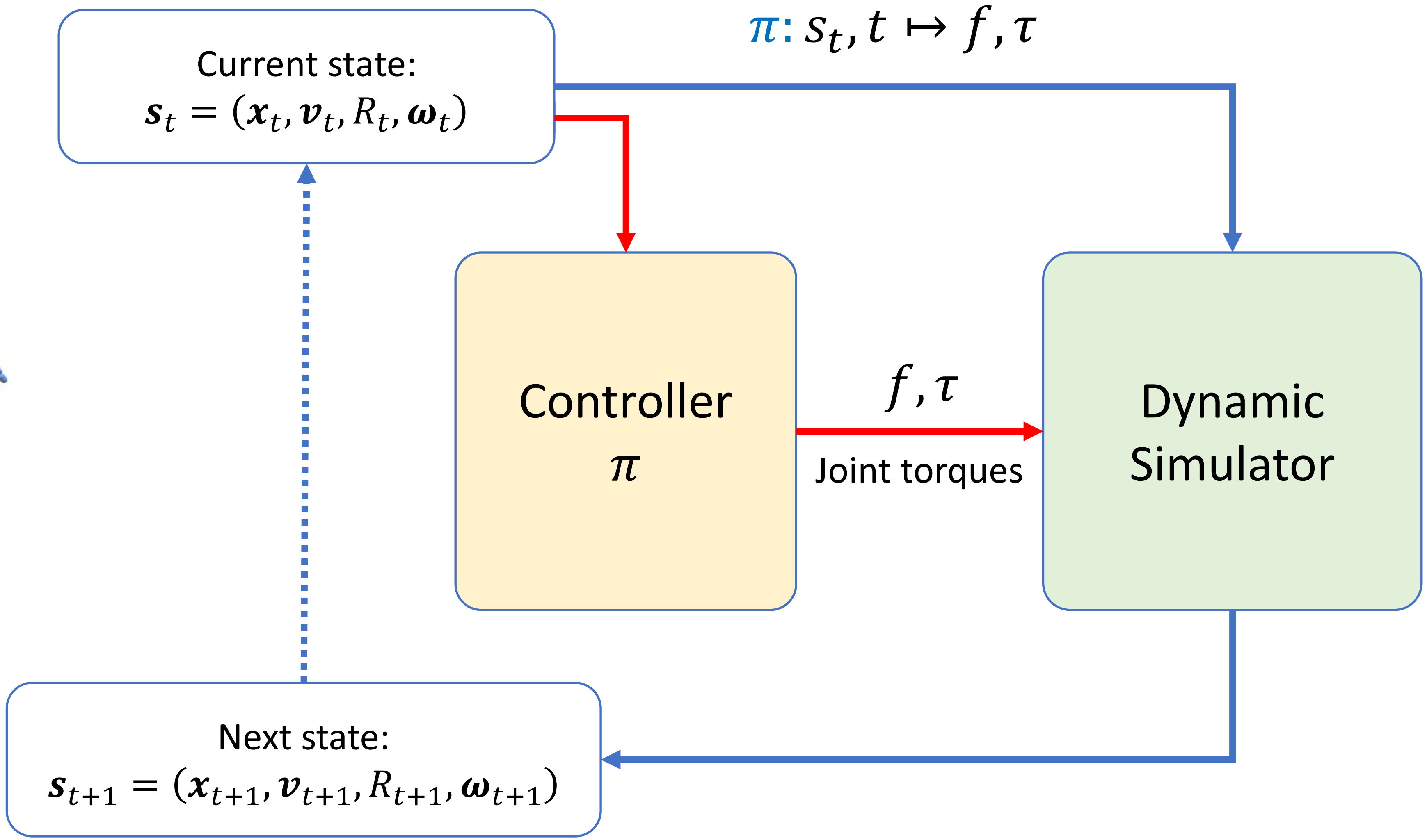
Underactuated



For many $[\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{v}}]$, there is no such $\boldsymbol{f}$ that produces the motion

Simulating + Control





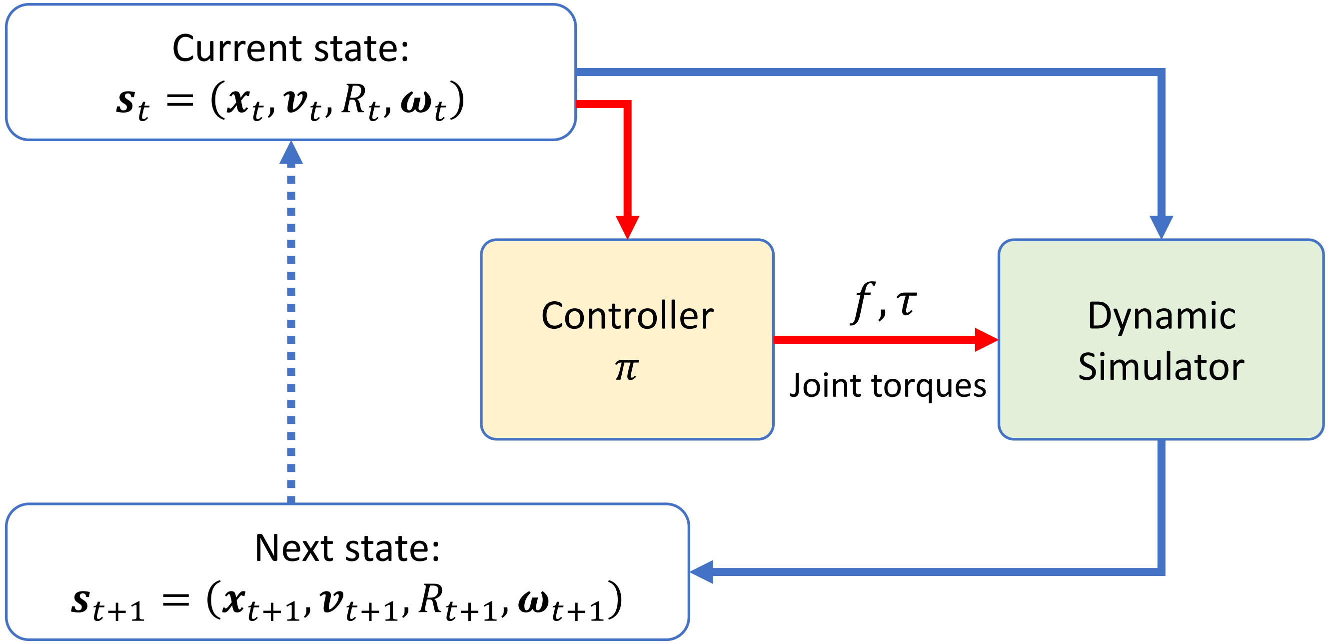
Feedforward vs. Feedback

Feedforward control:

$$f, \tau = \pi(t)$$

- Apply predefined control signals without considering the current state of the system
- Assuming unchanging system. Perturbations may lead to unpredicted results

$\pi: s_t, t \mapsto f, \tau$



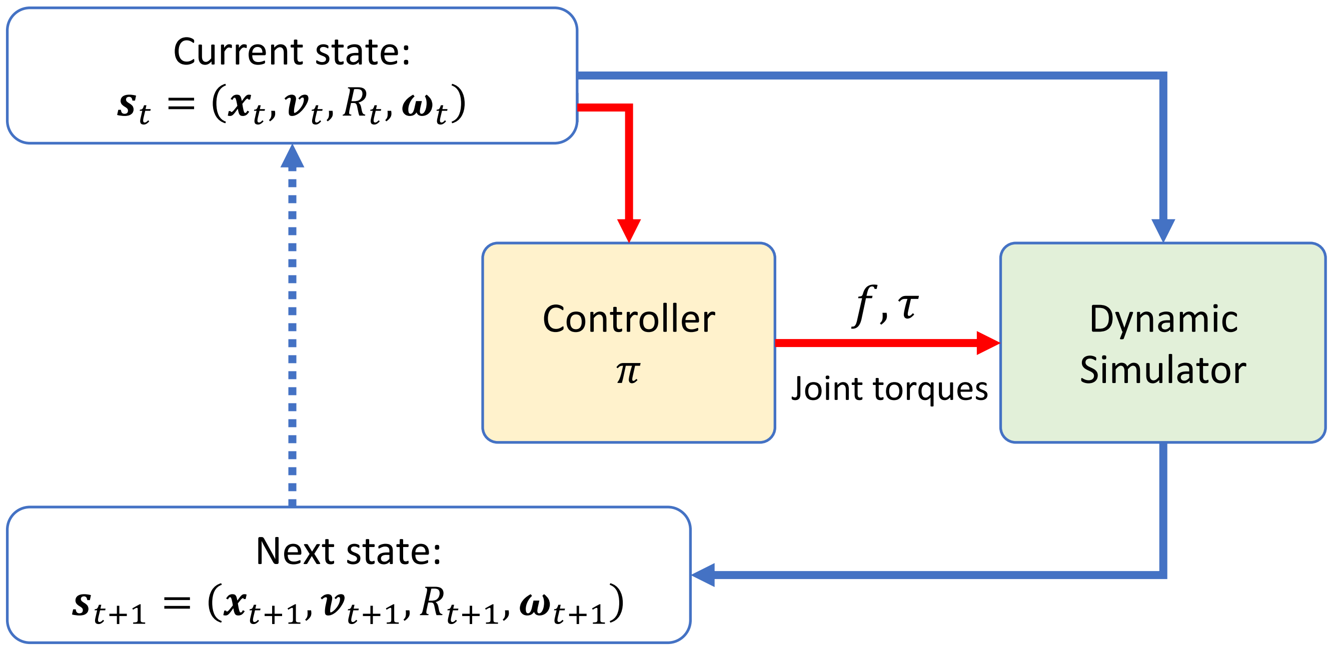
Feedforward vs. Feedback

Feedback control:

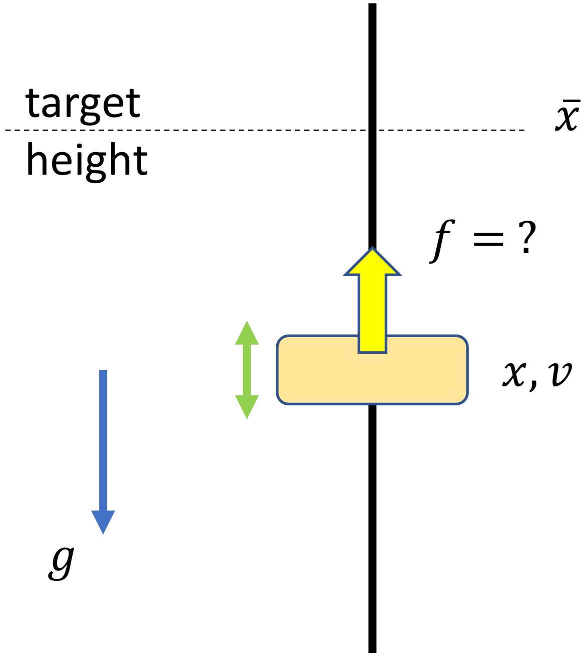
$$f, \tau = \pi(s_t, t)$$

- Adjust control signals based on the current state of the system
- Certain perturbations are expected. The feedback signal will be used to improves the performance at the next state.

$\pi: s_t, t \mapsto f, \tau$

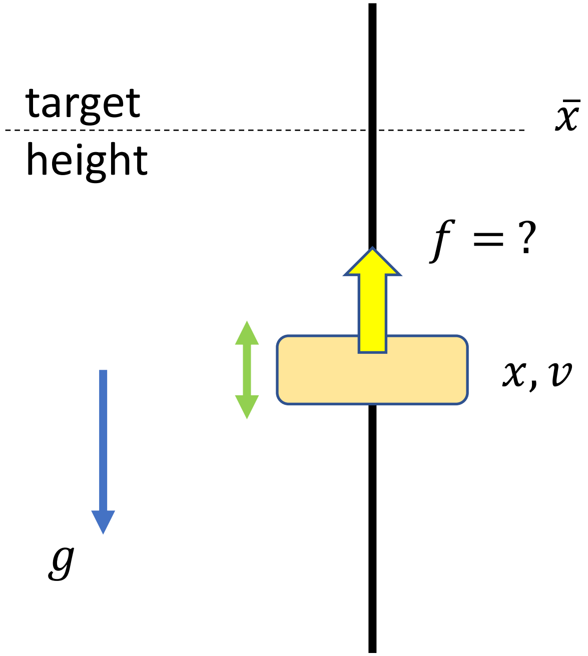


Proportional-Derivative Control



Compute force f to move
the object to the target height

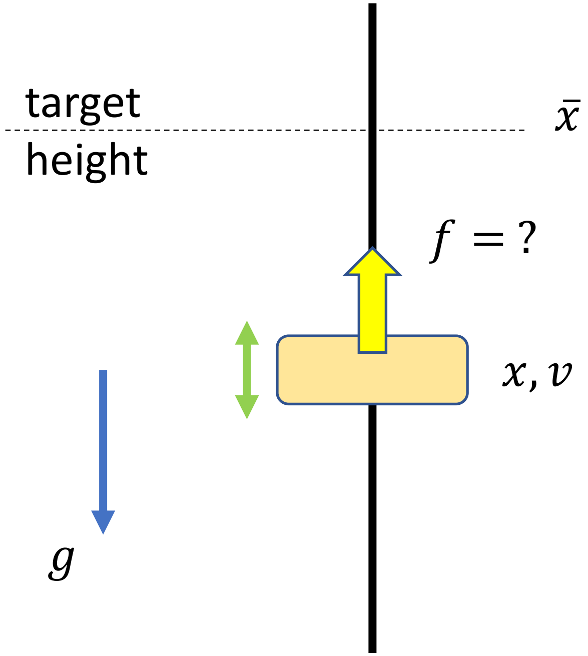
Proportional-Derivative Control



Compute force f to move
the object to the target height

$$f = k_p(\bar{x} - x)$$

Proportional-Derivative Control



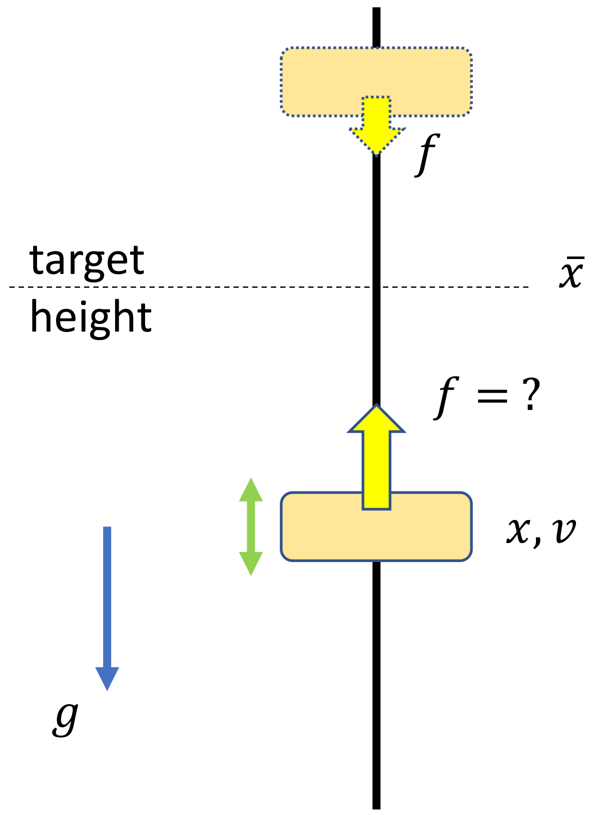
Compute force f to move
the object to the target height

target state

$$f = k_p(\bar{x} - x)$$

stiffness (gain) — current state

Proportional-Derivative Control



Compute force f to move
the object to the target height

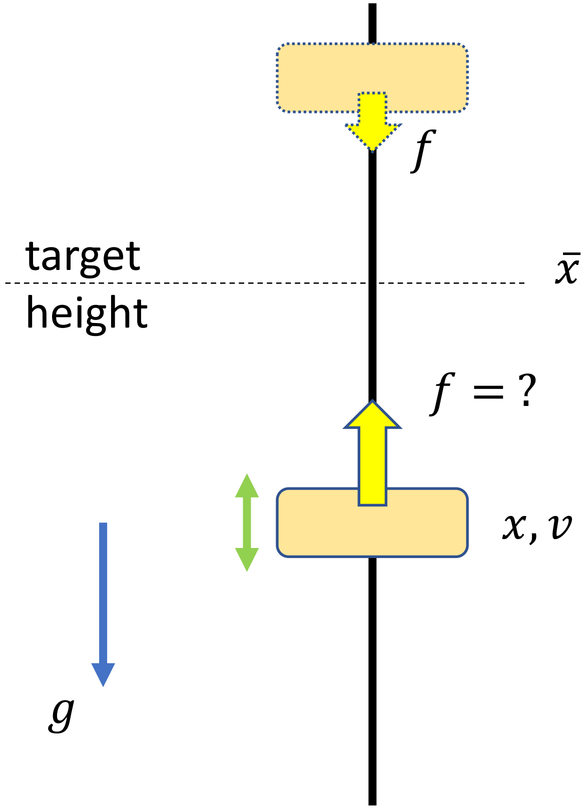
target
state

$$f = k_p(\bar{x} - x)$$

stiffness
(gain)

current
state

Proportional-Derivative Control



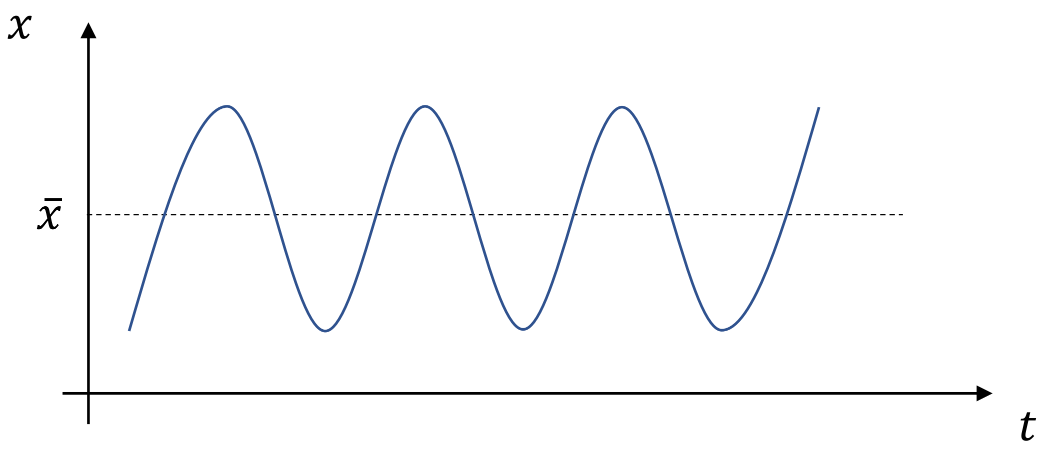
Compute force f to move the object to the target height

target state

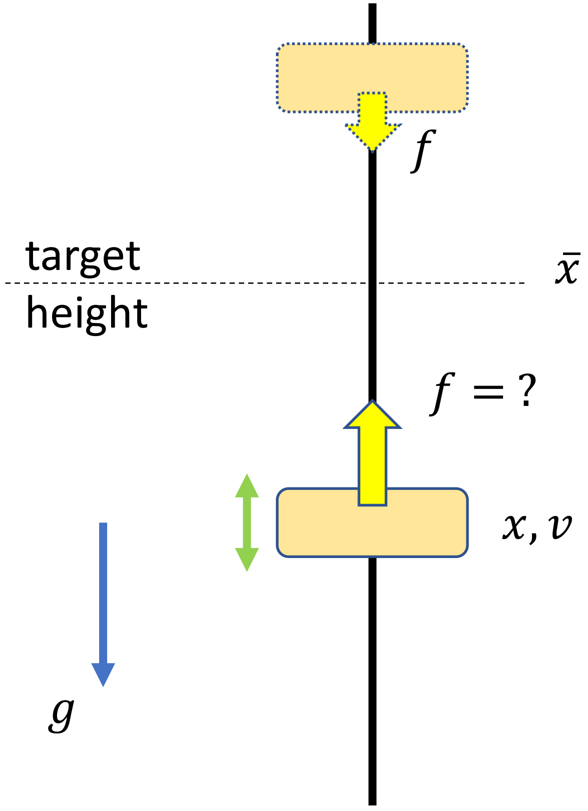
$$f = k_p(\bar{x} - x)$$

stiffness (gain)

current state



Proportional-Derivative Control

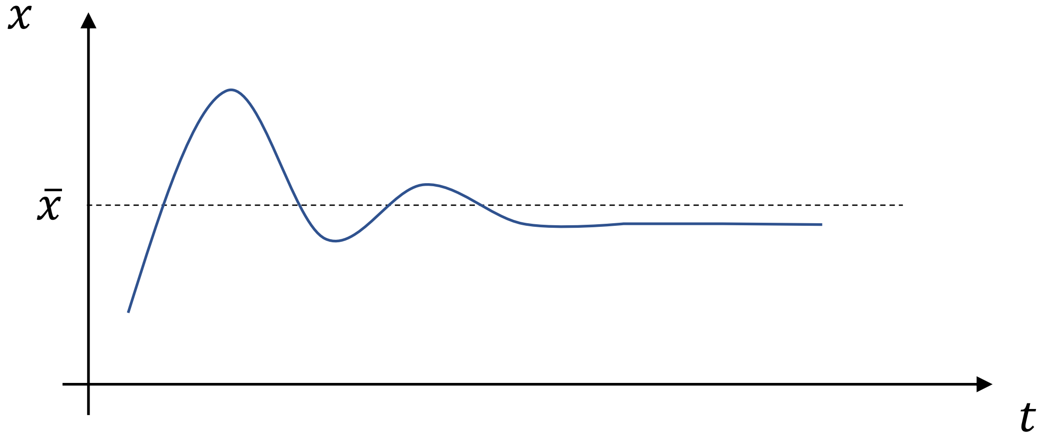


Compute force f to move
the object to the target height

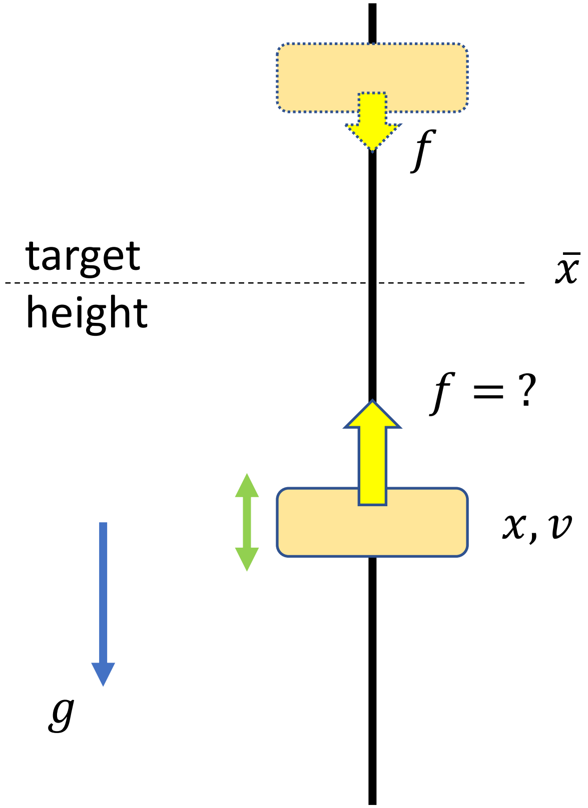
target state, damping

$$f = k_p(\bar{x} - x) - k_d v$$

stiffness (gain), current state, current velocity



Proportional-Derivative Control



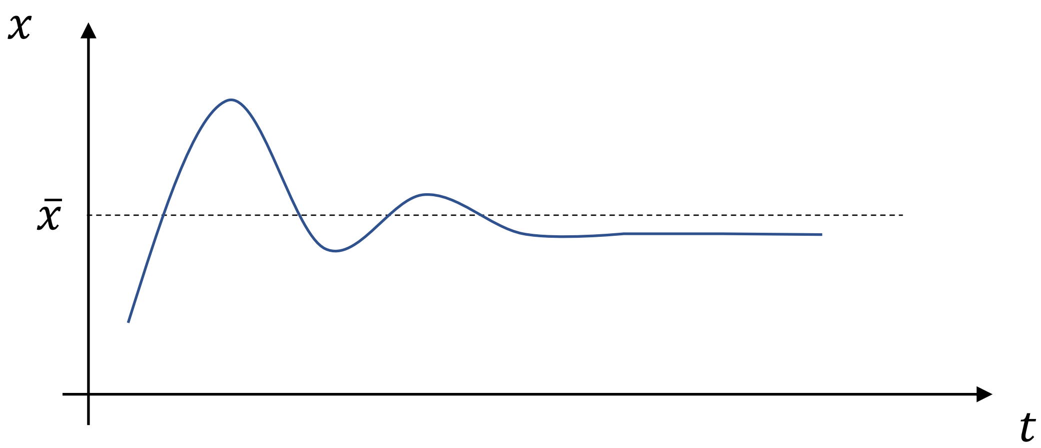
Compute force f to move
the object to the target height

Let $e = \bar{x} - x$ be the error

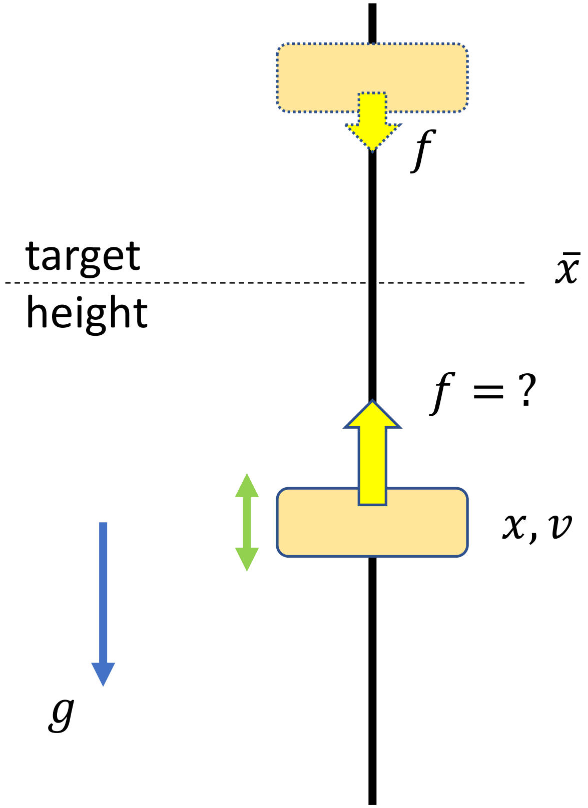
$$f = k_p e + k_d \dot{e}$$

proportional control

derivative control



Proportional-Derivative Control



Compute force f to move the object to the target height

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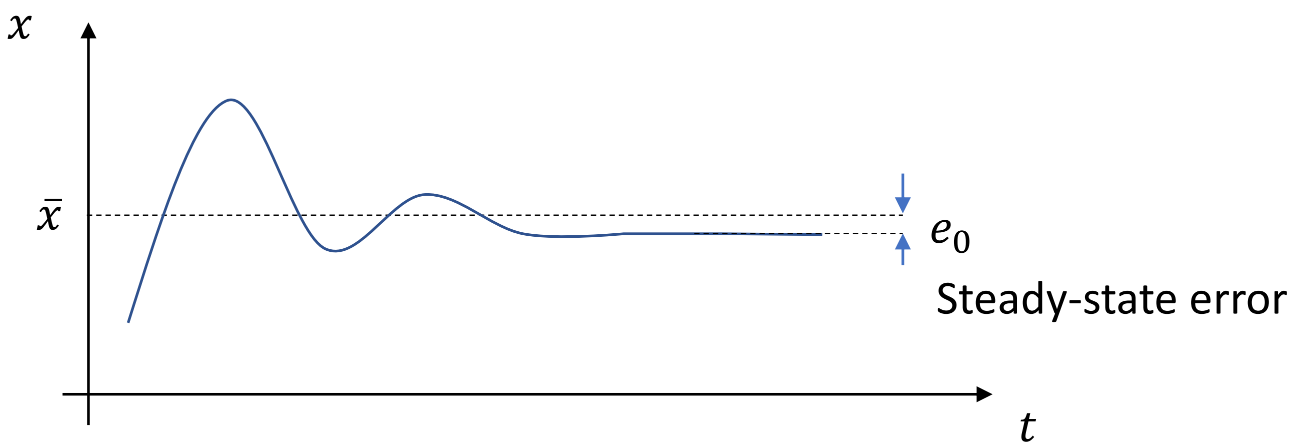
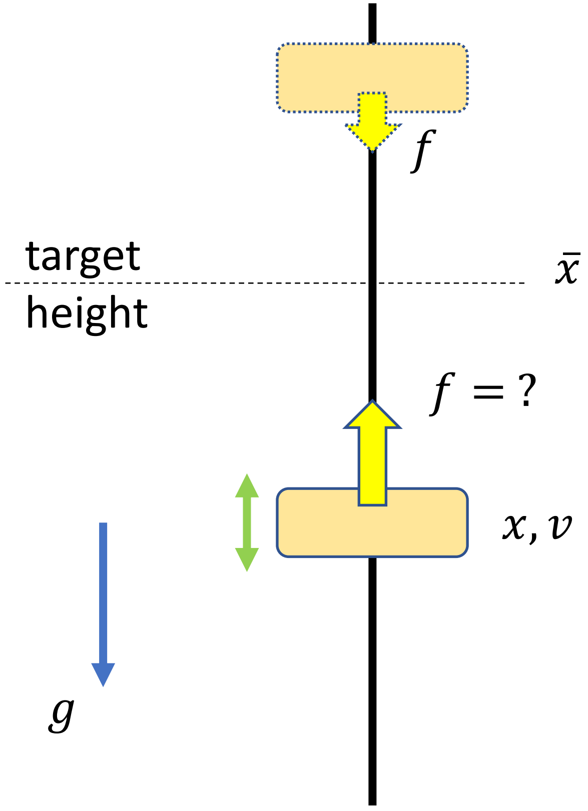
$$f = k_p e + k_d \dot{e}$$

proportional control

derivative control



Proportional-Derivative Control



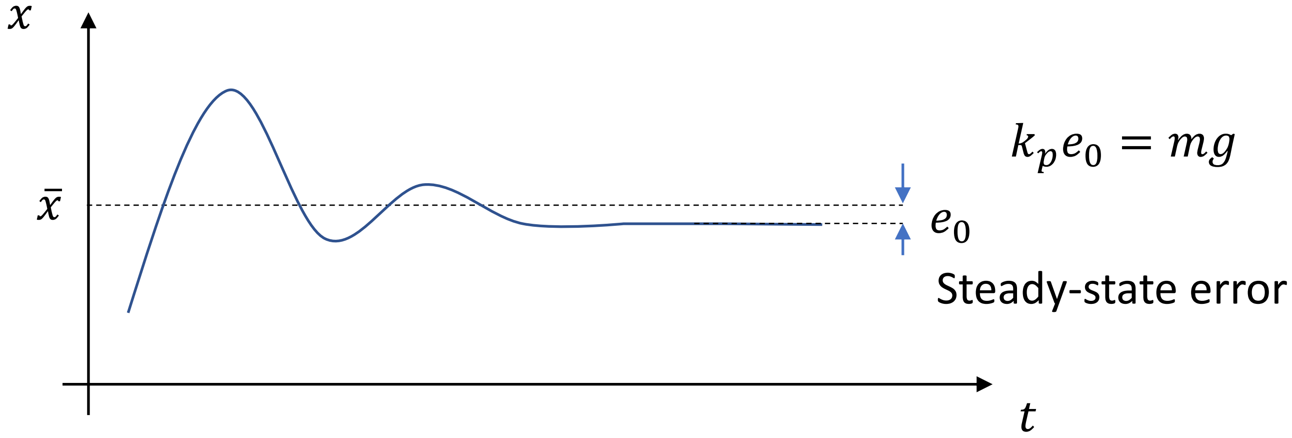
Compute force f to move
the object to the target height

Let $e = \bar{x} - x$ be the error

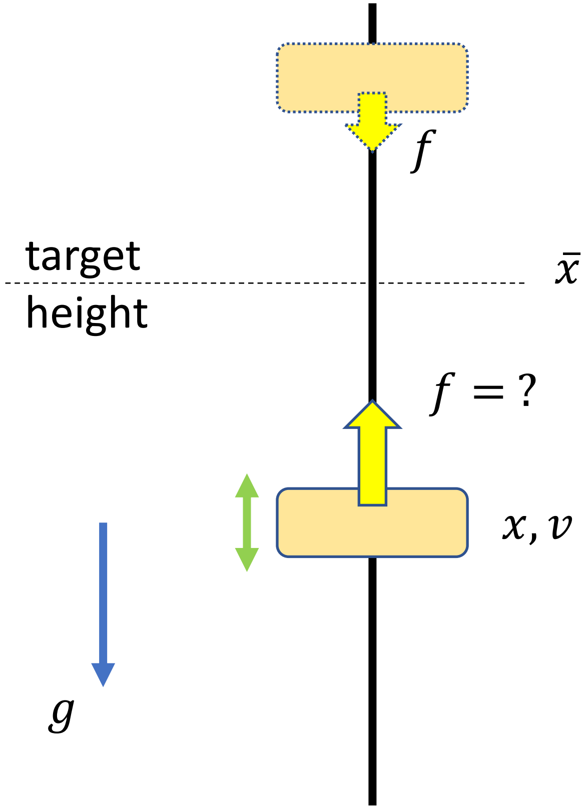
$$f = k_p e + k_d \dot{e}$$

proportional control

derivative control



Proportional-Derivative Control



Compute force f to move
the object to the target height

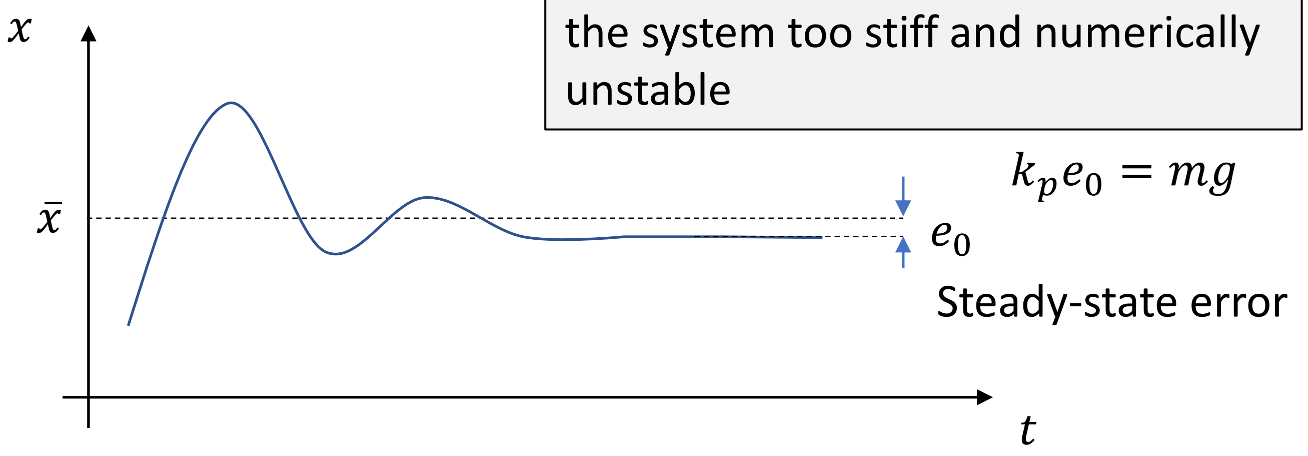
Let $e = \bar{x} - x$ be the error

$$f = k_p e + k_d \dot{e}$$

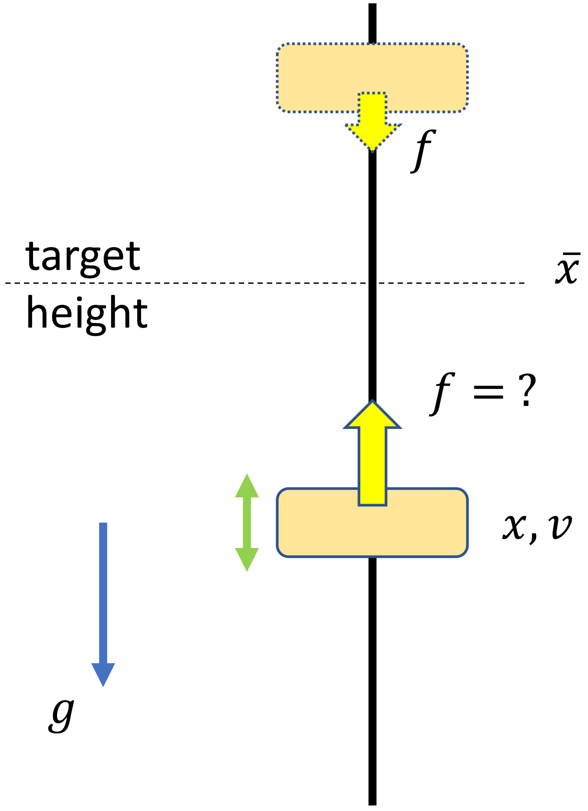
proportional control

derivative control

Increase stiffness k_p reduces the steady-state error, but can make the system too stiff and numerically unstable



Proportional-Integral-Derivative controller

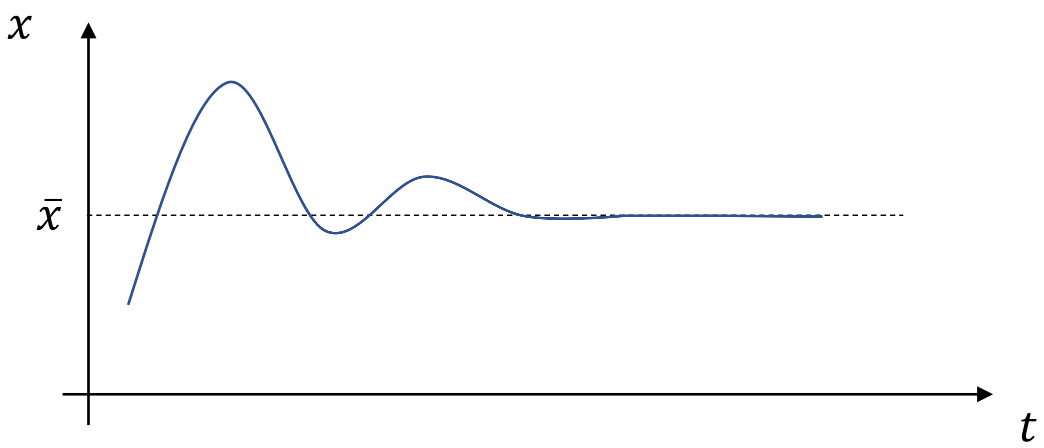


Compute force f to move the object to the target height

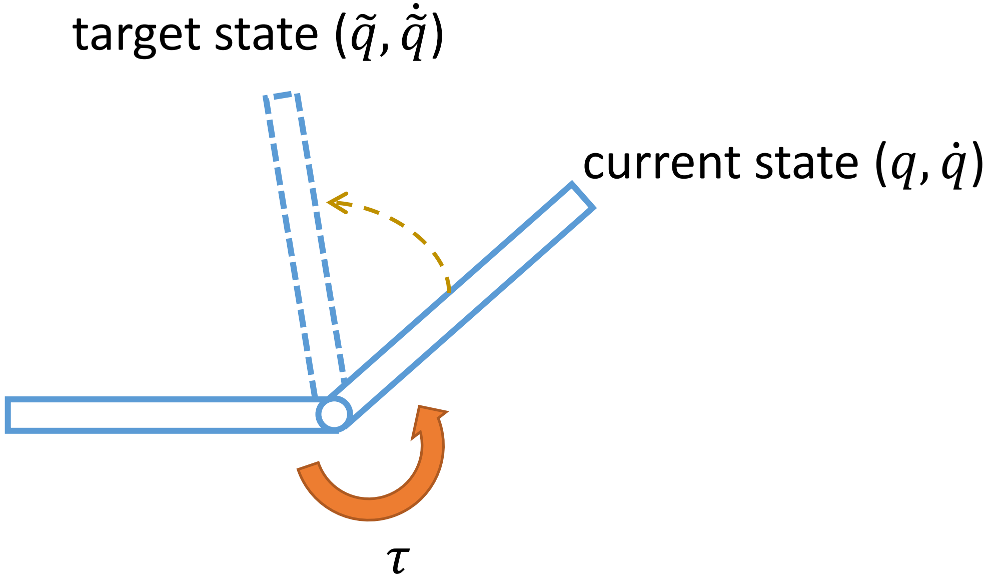
Let $e = \bar{x} - x$ be the error

$$f = k_p e + k_d \dot{e} + k_i \int_t e dt$$

proportional control | derivative control | Integral control (rarely used in animation)



PD Control for Characters



target velocity
often zero

target pose

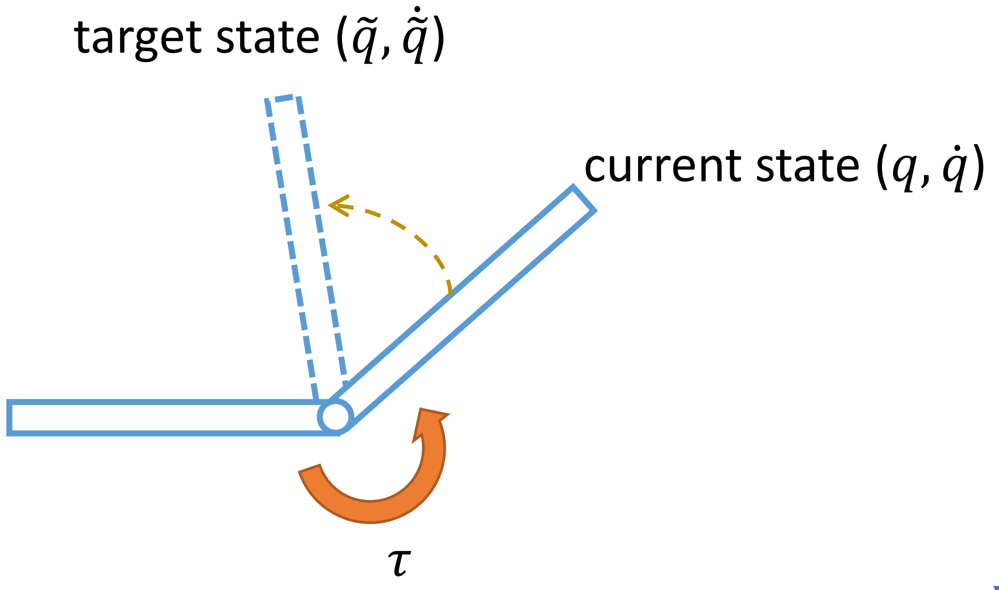
$$\tau = k_p(\bar{q} - q) + k_d(\dot{\bar{q}} - \dot{q})$$

joint torque

proportional control

derivative control

PD Control for Characters



target pose

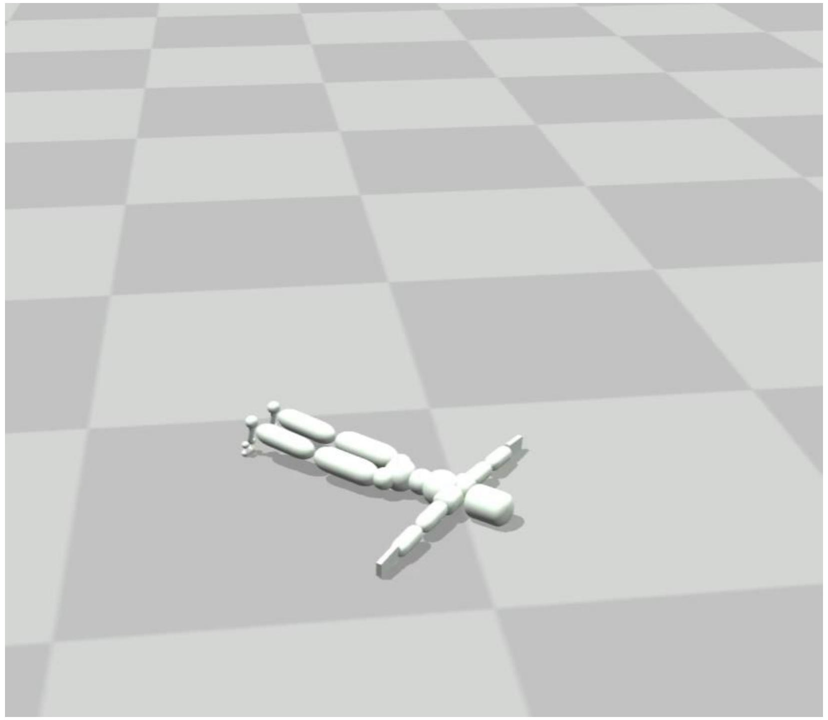
$$\tau = k_p(\bar{q} - q) - k_d\dot{q}$$

joint torque

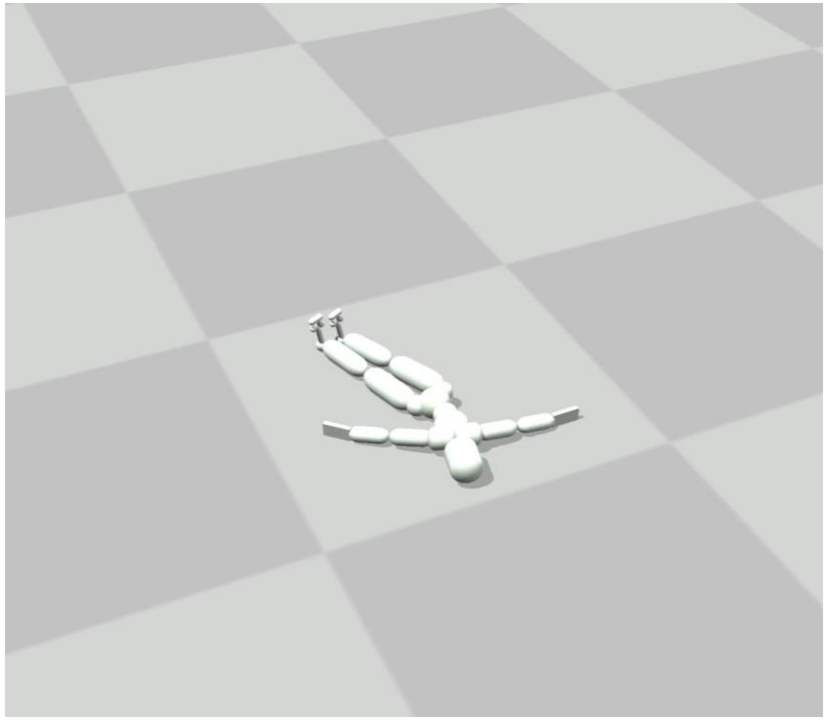
proportional control

derivative control

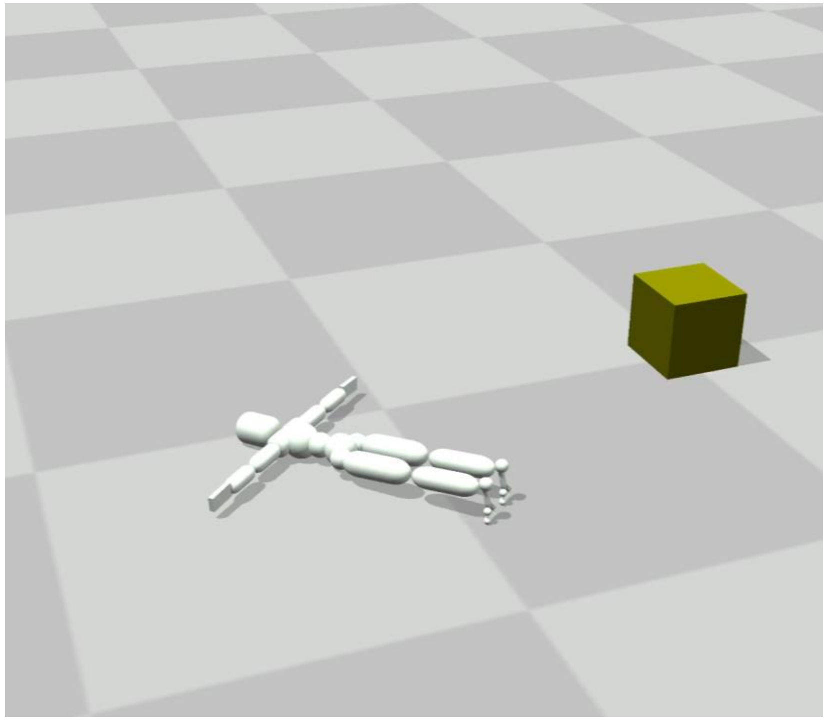
PD Control for Characters



Normal stiffness k_p

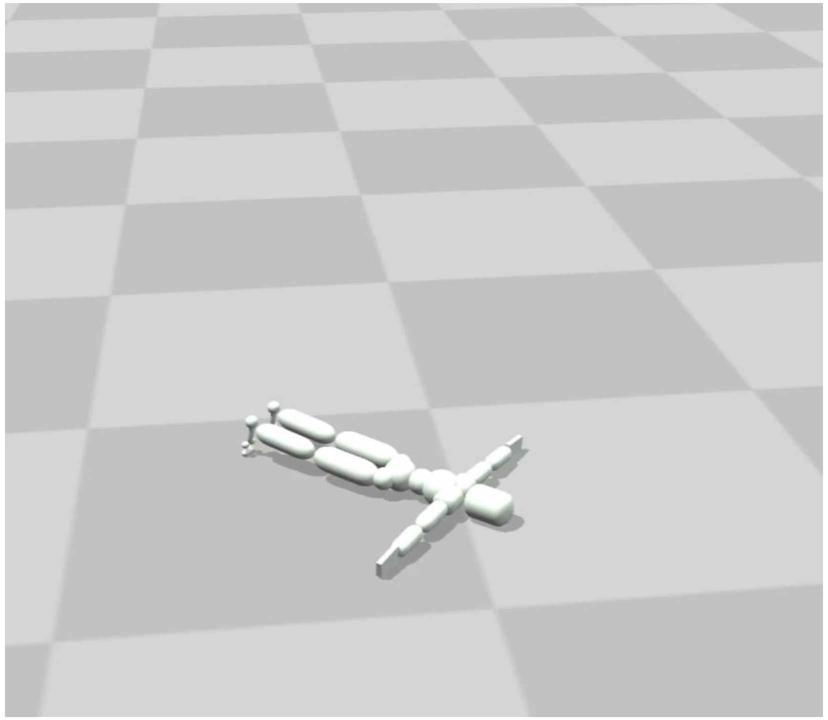


Small stiffness k_p

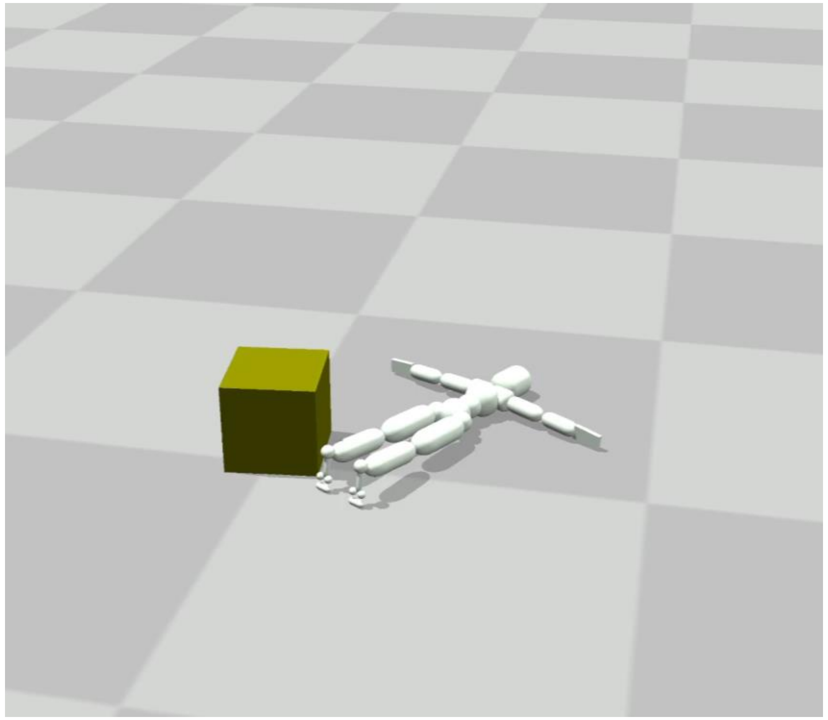


Large stiffness k_p

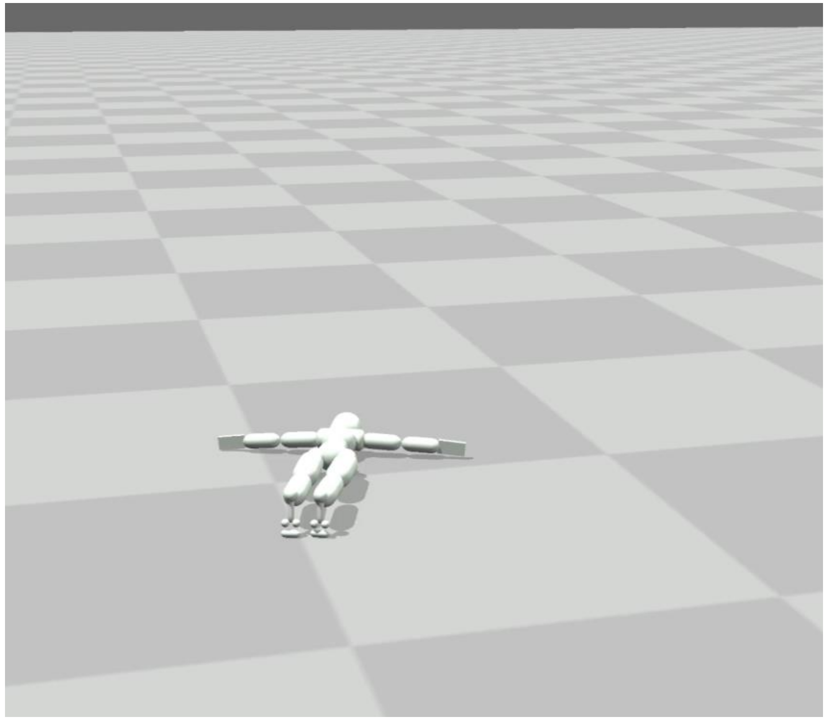
PD Control for Characters



Normal damping k_d

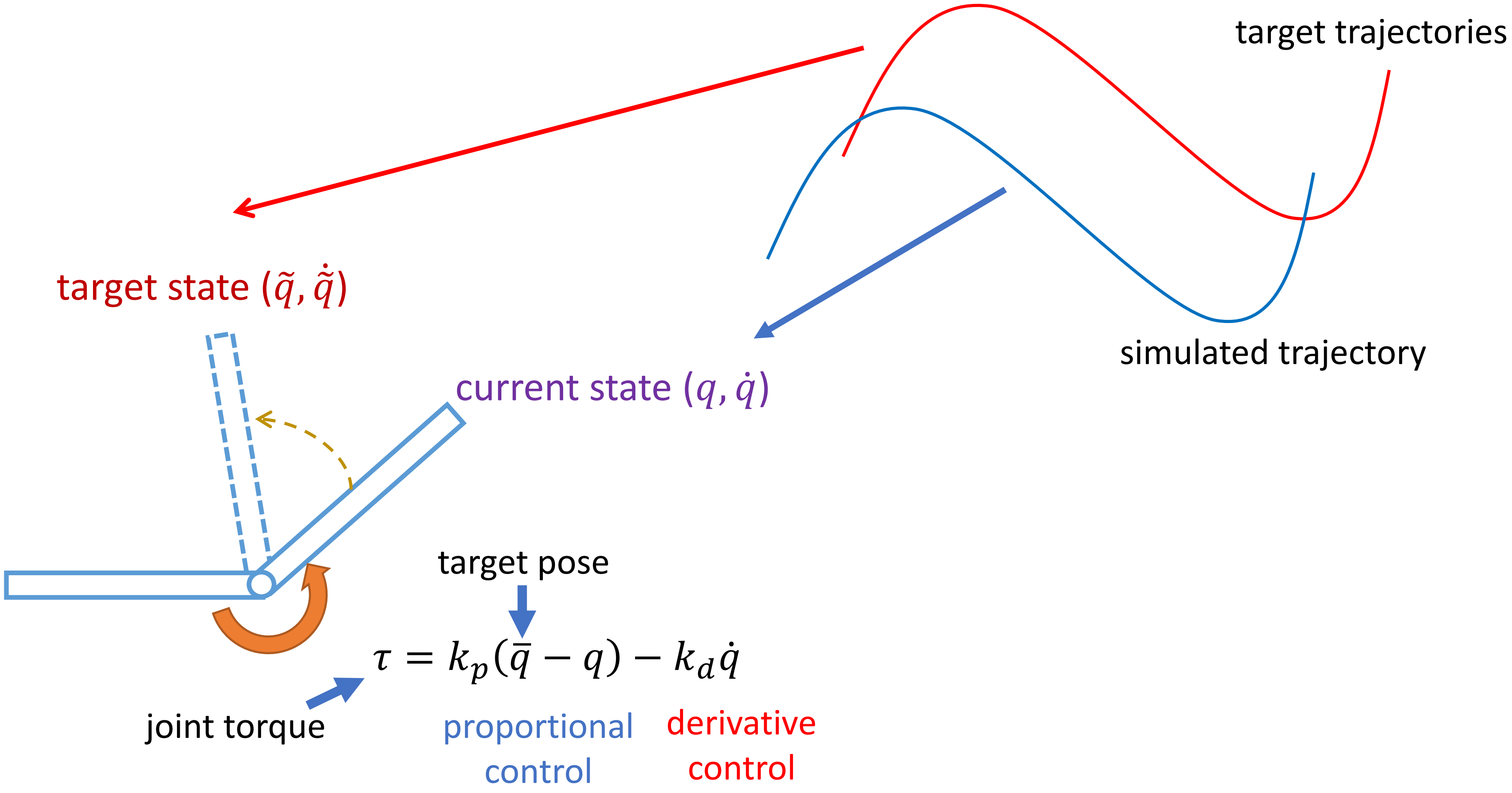


Small damping k_d

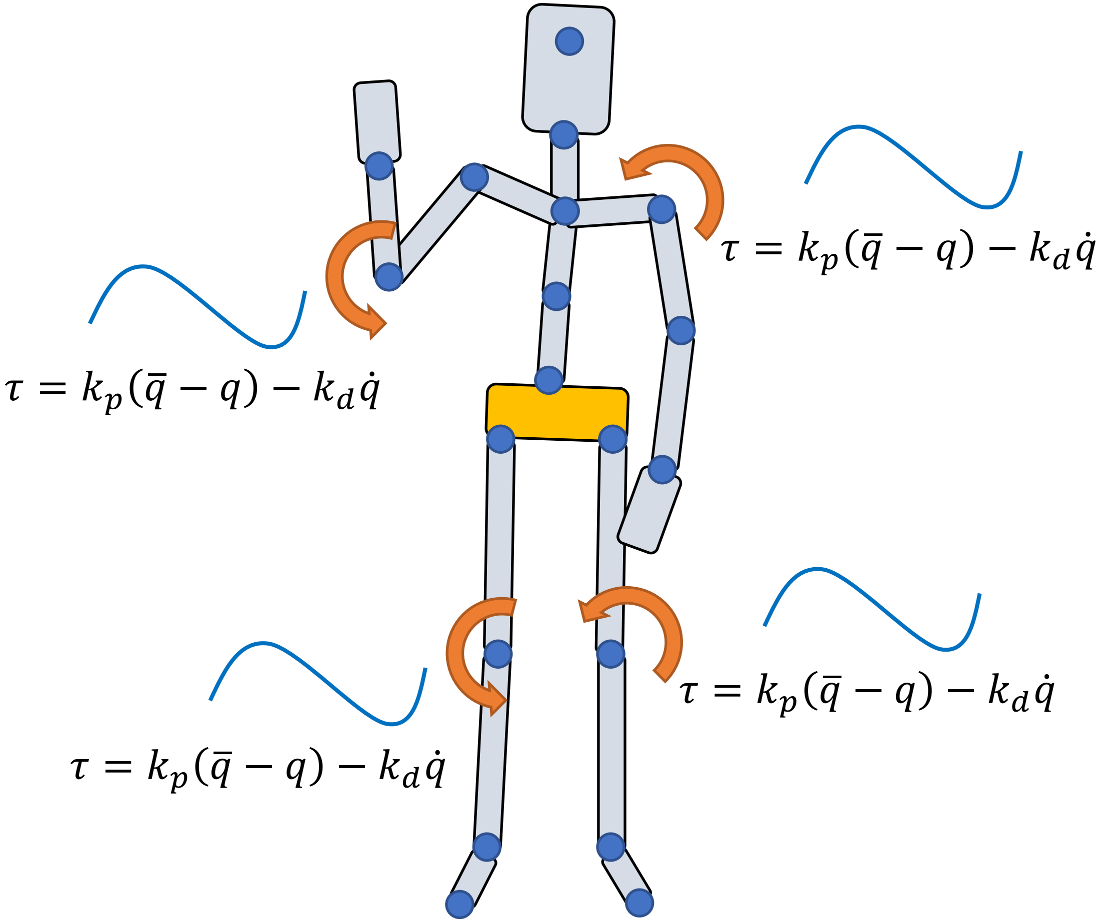


Large damping k_d

Tracking Controllers



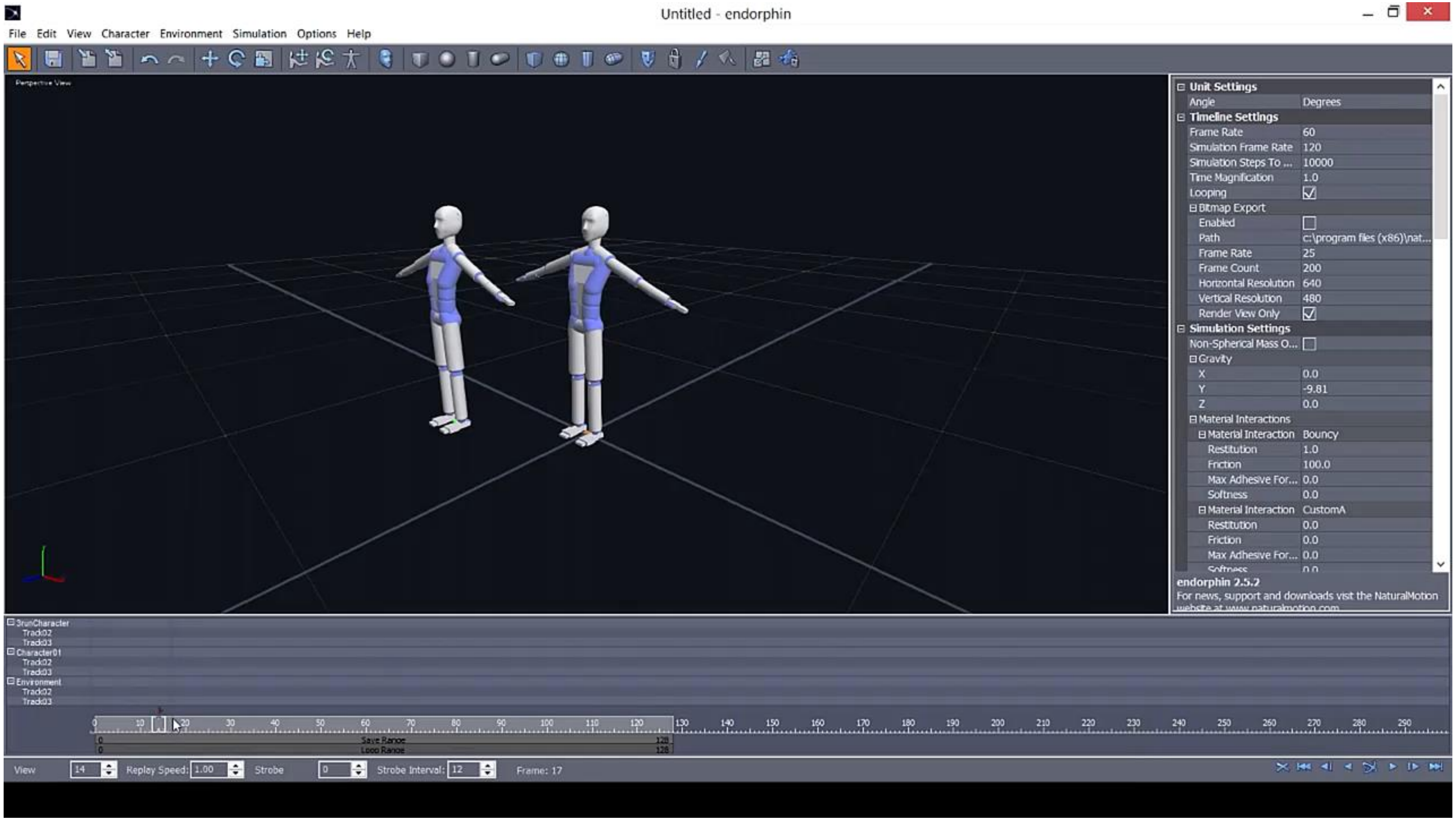
Full-body Tracking Controllers



Tracking a Trajectory

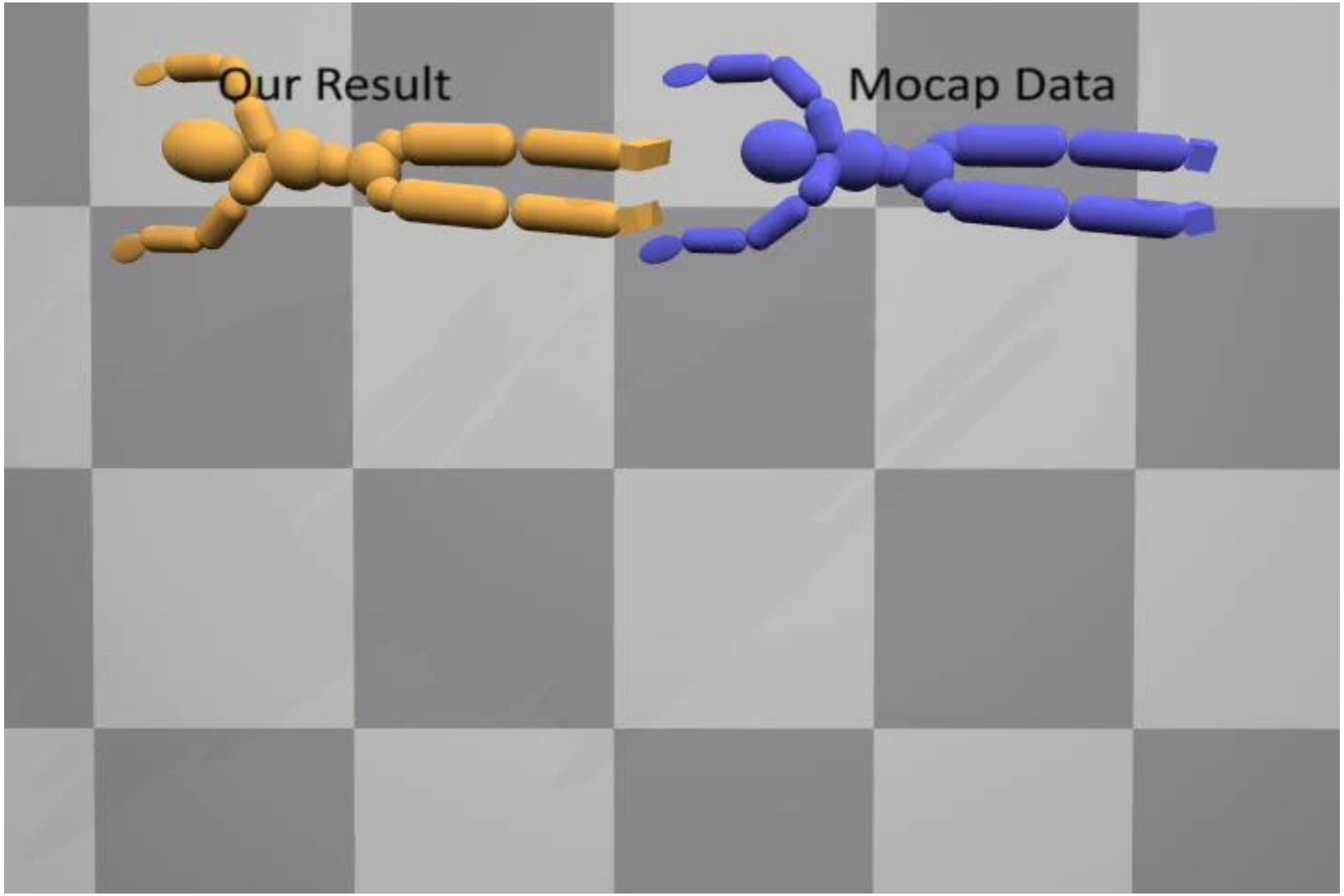
[Hodgins and Wooten 1995, Animating Human Athletics]

Trajectory Creation



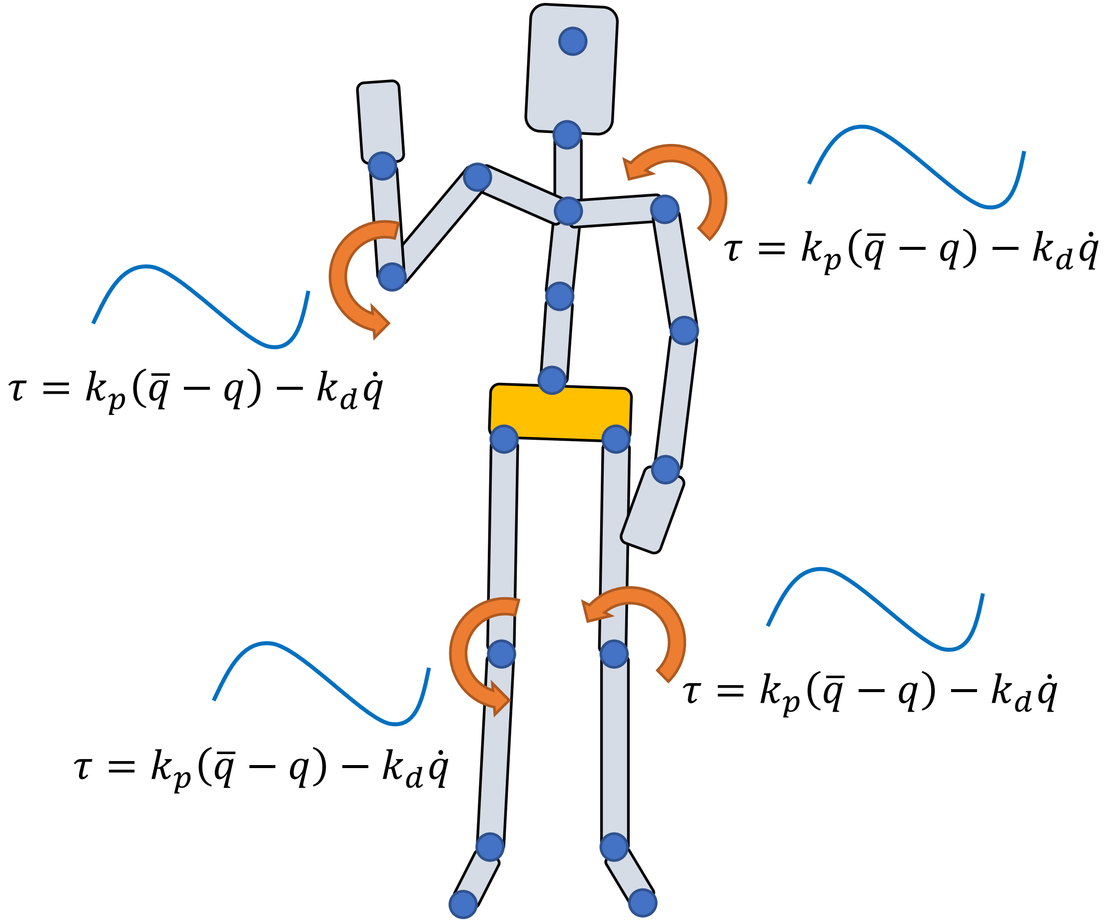
NaturalMotion - Endorphin

Tracking Mocap



[SAMCON – Liu et al 2010]

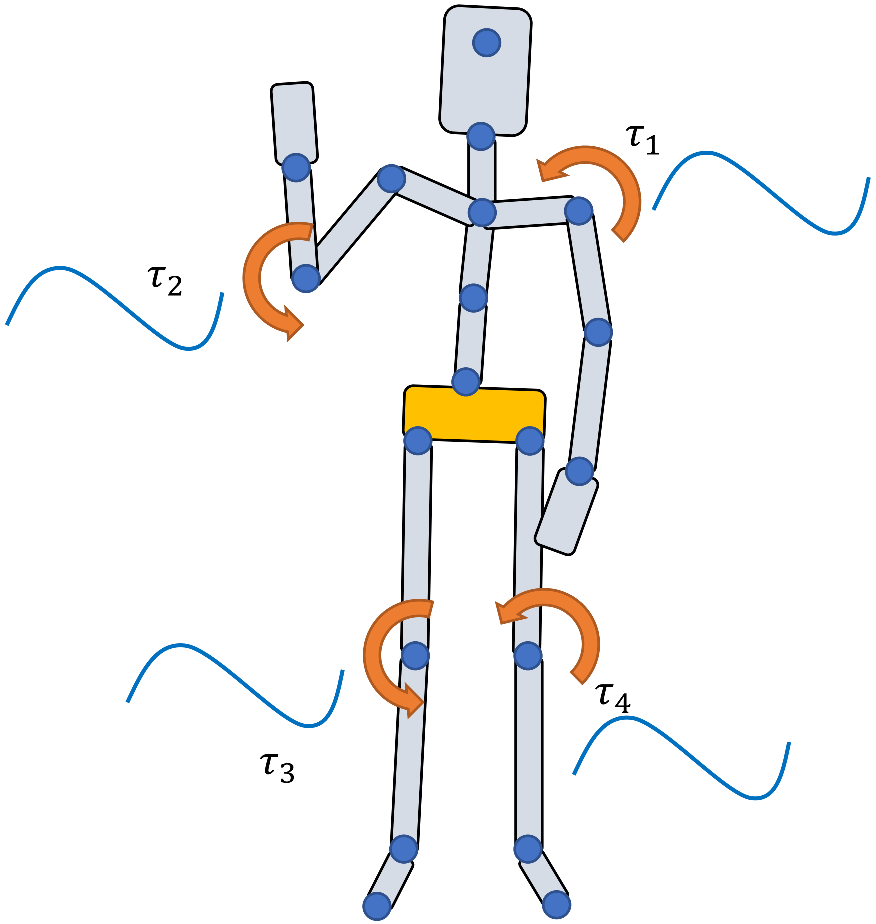
Full-body Tracking Controllers



Is PD control a feedforward control?

a feedback control?

Tracking Mocap with Joint Torques



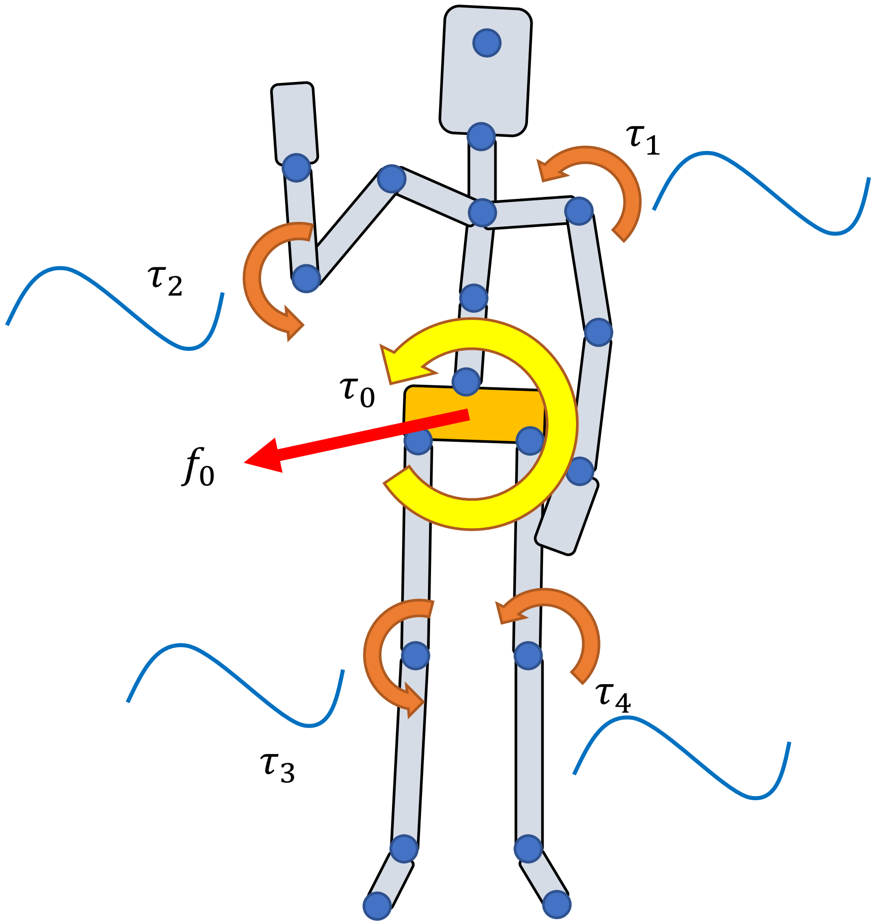
τ_j: joint torques

Apply τ_j to "child" body

Apply $-\tau_j$ to "parent" body

All forces/torques sum up to zero

Tracking Mocap with Root Forces/Torques



τ_j: joint torques

Apply τ_j to "child" body

Apply $-\tau_j$ to "parent" body

All forces/torques sum up to zero

f_0, τ_0: root force / torque

Apply f_0 to the root body

Apply τ_0 to the root body

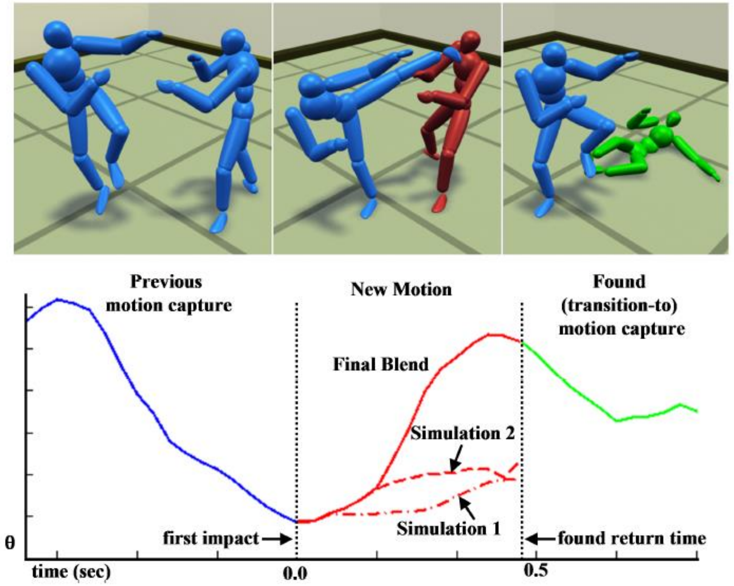
Non-zero net force/torque on the character!

Physically Plausible Animation

Party Animals

Totally Accurate Battle Simulator
https://www.youtube.com/watch?v=WFKGWfdG3bU

Mixture Simulation and Mocap



Zordan et al. 2005
Dynamic response for motion capture animation

Questions?